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Underwater Image Enhancement Method Based on Fuzzy Logic and Möbius Transformation

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***Abstract*—*Underwater images often suffer from low contrast, blurred details, and poor visibility due to light absorption and scattering. This paper proposes an enhancement method that integrates the membership value with two-stage Möbius transformations. The approach effectively expands gray-level distribution, restores fine details, and maintains edge and structural integrity, while improving visibility in turbid conditions. Experimental results, supported by both qualitative observations and quantitative metrics (MSE and PSNR), demonstrate significant improvements in visual quality. The method is simple, easy to implement, and robust across varying levels of degradation, making it a practical solution for underwater image enhancement.*

***Keywords*— *Underwater image, image enhancement，fuzzy logic.*

I. Introduction

Underwater image enhancement (UIE) is fundamental to marine exploration, underwater robotics and computer vision applications. However, due to light absorption and scattering in water at different wavelengths, underwater images often have low contrast and colour distortion, and blurred details. This significantly degrades visual quality and limits the performance of downstream tasks such as object detection and scene understanding [1 – 3]. Therefore, developing effective and robust enhancement methods remains challenging.

The early research on image enhancement was mainly about Retinex-based methods and illumination-reflection decomposition models. Improved Retinex algorithms combined with guided filtering and adaptive histogram equalisation, for example, have demonstrated effectiveness in enhancing brightness, suppressing noise and preserving edge details [4]. These methods have also been adapted for use in underwater scenarios by incorporating colour correction and illumination estimation strategies to counteract colour cast and contrast degradation [5]. While such approaches are computationally efficient and physically interpretable, their performance is often limited by simplified assumptions and inadequate adaptability to complex underwater environments.

Non-physical model-based methods enhance images by exploiting statistical properties or applying transformation functions without explicitly modelling the imaging process. Representative techniques include histogram equalization, frequency-domain enhancement, and nonlinear mapping methods [6 – 7]. Nonlinear functions, including Gamma correction and S-shaped mappings, have been extensively utilised to modify intensity distributions and enhance contrast [7]. Furthermore, hybrid approaches that combine spatial-domain processing with multi-scale or fusion strategies have been proposed to improve robustness under varying degradation conditions [8]. However, these methods typically rely on heuristic parameter tuning and may introduce over-enhancement or visual artefacts when dealing with diverse underwater scenes.

In recent years, learning-based methods, in particular deep neural networks, have achieved significant advances in UIE. Convolutional neural network (CNN)-based models, such as UWCNN, have been shown to learn the mapping from degraded images to high-quality outputs in an end-to-end manner [1]. The advent of substantial, well-established benchmark datasets, such as UIEB, has further promoted quantitative evaluation and enhanced model generalisation [9]. In order to address the discrepancy between synthetic and real-world underwater data, domain adaptation techniques have been incorporated with a view to enhancing robustness across different environments [10]. Furthermore, advanced architectures integrating CNNs and Transformer mechanisms have demonstrated superior capability in capturing complex features and improving enhancement performance [11]. Despite their effectiveness, these methods often require large annotated datasets and substantial computational resources, and their limited interpretability restricts their applicability in real-world scenarios.

A recent survey of literature on the subject has systematically reviewed the development of underwater image enhancement techniques and highlighted key challenges and future directions [2 – 3,12]. The extant literature suggests a shift in research focus towards hybrid frameworks that integrate conventional enhancement methodologies with learning-based approaches. In particular, the integration of nonlinear mapping with fuzzy logic has demonstrated potential for managing uncertainty and spatially varying degradation in underwater images [8,13]. Nevertheless, achieving a balance among enhancement performance, computational efficiency, and model interpretability remains an open issue.

The observations presented herein have motivated the development of an effective underwater image enhancement framework that leverages the advantages of nonlinear modelling and adaptive enhancement mechanisms. It is hypothesised that the proposed approach will enhance visual quality while preserving computational efficiency and interpretability, thus offering a pragmatic solution for real-world underwater vision applications.

The proposed method is an underwater image enhancement technique that uses membership functions and two Möbius transformations. This technique involves simple, easy-to-implement steps. Experimental results demonstrate that the proposed method can enhance the clarity of underwater images.

II. Proposed Method and Experiments

A. Proposed Method

This subsection presents a method for enhancing images that combines fuzzy membership functions with a two-stage modification strategy. First, the membership degree of each pixel is calculated. Then, the membership values are adjusted twice using Möbius transformations. Finally, the modified results are mapped back to the desired grey-level domain to produce the enhanced image. The steps are as follows:

Calculate membership values by converting the grayscale values of the input image from the range [0, 255] to [0, 1] .

$$u_{ij} = \frac{x_{ij} - x_{\min}}{x_{\max} - x_{\min}}, \tag{1}$$

Where x_{ij} is the pixel value, and $x_{\max}$ and $x_{\min}$ are the highest and lowest grayscale values in the image, respectively.

Modified using the Möbius transformation,

$$u'_{ij} = \frac{au_{ij} + b}{cu_{ij} + d}, \ a,b,c,d \in C. \tag{2}$$

Second Modification,

$$u''_{ij} = \frac{au'_{ij} + b}{cu'_{ij} + d}, \ a,b,c,d \in C. \tag{3}$$

Restore image,

$$y_{ij} = 255 \cdot u''_{ij} \cdot (u''_{\max} - u''_{\min}) + u''_{\min}, \tag{4}$$

where $u''_{\max}$ and $u''_{\min}$ are the highest and lowest value of x_{ij} .

B. Experiments

In the experiment, set $a = -1$, $b = c = d = 1$, the test images include 8 underwater grayscale images. Following the enhancement of the images, the experiments were subjected to evaluation using the Mean square error (MSE) and Peak signal to noise ratio (PSNR)[14,15] .

$$MSE = \frac{1}{m \times n} \sum_{i=1}^{m} \sum_{j=1}^{n} (x_{ij} - y_{ij})^2, \tag{5}$$

$$PSNR = 10 \times \log_{10} \frac{(2^n - 1)^2}{MSE} dB, \tag{6}$$

The experimental results (see Fig. 1 and Tab. 1) demonstrate that original underwater images are characterised by low contrast, blurred details, and poor visibility, which is attributable to light absorption and scattering. Following the enhancement process, there was a substantial improvement in visual quality, a finding corroborated by both MSE and PSNR evaluations.

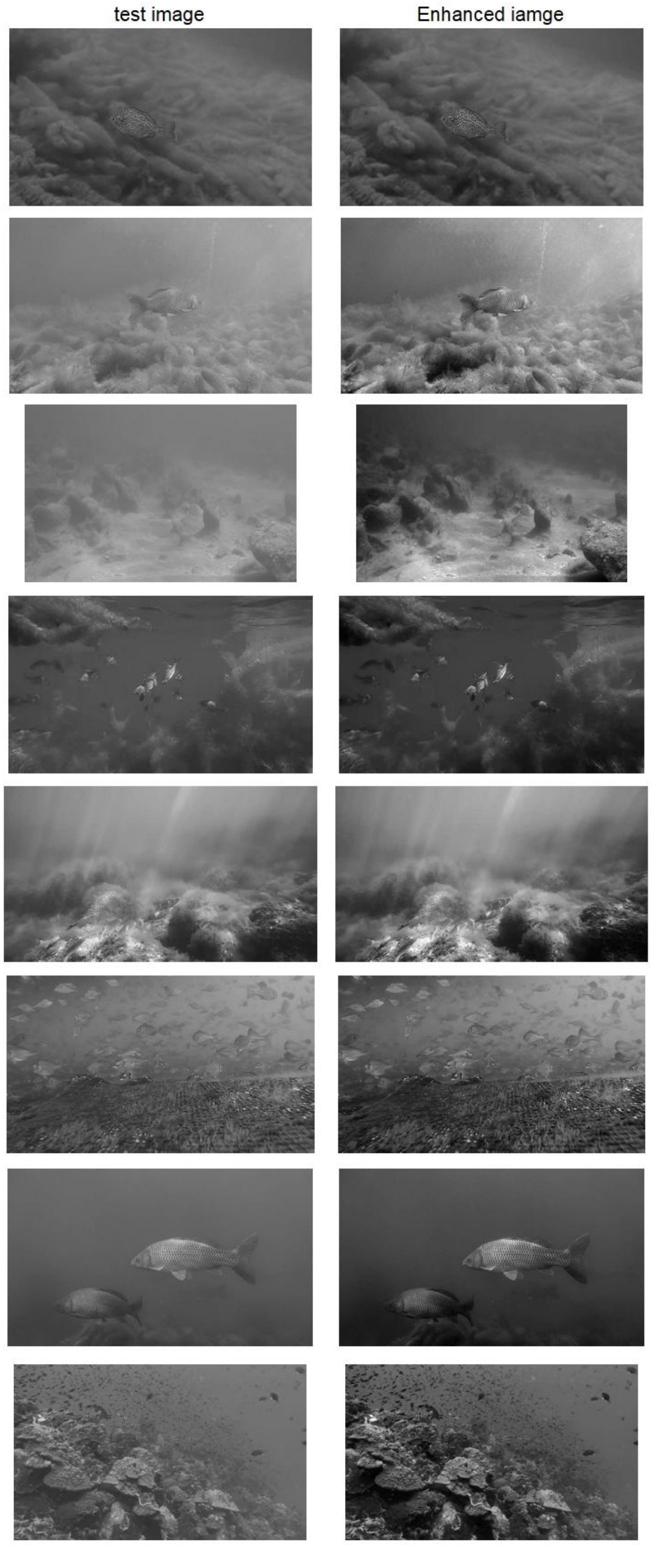

Fig. 1. Test images and enhanced images.

The PSNR values range from 16.41 dB to 34.04 dB, while MSE varies from 25.63 to 1484.96. Lower MSE and higher PSNR (e.g., 25.63/34.04 dB and 58.43/30.46 dB) indicate superior reconstruction quality, while satisfactory results are maintained under severe degradation, thereby demonstrating the system's robustness.

TABLE 1. MSE and PSNR values for the test and enhanced images

MSE	PSNR
25.63	34.04
312.93	23.18
1484.96	16.41
413.63	21.96
58.43	30.46
122.58	27.25
776.30	19.23
527.44	20.91

The method has been demonstrated to be effective in expanding the grey-level distribution, enhancing contrast and improving foreground-to-background separation. Furthermore, it has been demonstrated that this approach facilitates the restoration of fine details and textures, thereby suggesting that fuzzy membership modelling preserves intrinsic information and that the two-stage Möbius transformation refines local features.

Furthermore, the enhanced images exhibit clear edges and structural continuity, while reducing haze and enhancing visibility in turbid conditions. Notwithstanding the occurrence of slight over-enhancement and minor noise amplification, the method as a whole attains an optimal equilibrium between contrast enhancement, detail preservation, and structural integrity.

III. Conclusions

The proposed methodology integrates the rate-degree function with Möbius transformations to enhance underwater images. The experimental results indicated a notable enhancement in image contrast. The method is characterised by its simplicity and ease of implementation, which renders it a valuable asset.

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