International Journal of Multidisciplinary Research and Publications
ISSN (Online): 2581-6187

Survey on Contemporary Methods for Power System Voltage Stability Evaluation

Peter Obinna Mbamaluikem[1], Oluwaseun Ibrahim Adebisi[2], Kamoli Akinwale Amusa[3]
[1]Department of Electrical/Electronics Engineering, Federal Polytechnic Ilaro, Ogun State, Nigeria.
[2, 3]Department of Electrical and Electronics Engineering, Federal University of Agriculture, Abeokuta, Nigeria.
Email address: peter.mbamaluikem@federalpolyilaro.edu.ng

Abstract**—***Voltage instability remains a major cause of large-scale blackouts, and it still poses significant threats to the stability of modern electric power systems. The paper provides a quantitative and technically accurate overview of the definitions of voltage stability, classification models, and the latest evaluation methods. Special focus is made on the ability of the voltage stability analysis to detect weak, unstable, and poorly manageable areas within transmission networks before commencing cascading failures. Based on the most popular high-impact archives, the paper classifies the assessment methodologies into static, dynamic, and measurement-based models, discussing their mathematical background, analysis utilities, and applicability of the operational aspects in practice. Comparative analyses can be used to source the performance trade-offs between offline, online, and integrated implementations with a complementary elaborated discussion of the differences between the static and dynamic stability testing with respect to objectives, modelling requirements, and the time-domain properties. This review also points out a gap in the existing research, such as the inclusion of the synchrophasor-based monitoring, the use of data-driven learning models, and adaptive mixed-format systems to assess stability in real-time. The obtained results provide a systematic technical reference to the researcher, utility operators, and system planners seeking to make the grid more resilient and reliable.*

Keywords**—** *Voltage stability assessment, Power system stability, Static stability, Dynamic voltage stability, Voltage stability indices, Power system network.*

I. INTRODUCTION

Today, with the population growth, industrialization, and the necessity to reduce the impact of climate change by cutting carbon emissions, the traditional centralized generation of electrical power is not adequate to meet current electricity demand and solve problems of providing clean, affordable, reliable, and sustainable electrical power [1]. Thus, most developed and developing countries are utilizing different sources of energy in an attempt to solve their electrical power generation issues [2, 3]. These advancements have indeed significantly stimulated several dynamic behaviors of the power network, thus affecting the system's behaviour upon exposure to disturbances such as faults, various loading changes, massive loading losses, and generation losses. These faults, natural or man-made, stress the transmission system, which may result in electrical power transmission system instability [4-6]. This has affected the electrical power generation and utilization techniques worldwide. [7-9]. These conditions also boosted the decentralization of electricity, producing inevitable revolutionary adjustments in how electrical power transmission lines were structured, rendering long-distance bulk energy transmission a demanding requirement [10, 11]. The current state of electricity generation and utilization compels power systems to run at the border of the voltage stability margin, and is prone to system collapse on a large scale [12, 13], unless monitored closely.

Moreover, with increased demand for electricity, there is a corresponding increase in loading of transmission lines, which results in the problem of voltage instability [14-15], particularly on weak grids that possess lower supply security and a set of unsatisfactory and inflexible generation capacity. If bulk powers are to be transferred through these AC corridors, system voltage stability will be compromised [16-20]. This fact affirms why, in the majority of developing and underdeveloped countries, the grids are run close to or sometimes above the voltage stability margins [21].

The voltage instability occurs when the power system fails to produce or provide the necessary reactive power to maintain the acceptable voltage profiles in the grid. This scenario results in disequilibrium between available reactive power and the demand for reactive power. In case the imbalance is not corrected, then this can trigger an immediate drop in system voltage. Following this, power system stability (PSS) shows how the system can recover a stable operating point during and after system disturbances, and maintain continuous customer service [22-23]. In this regard, stability analysis is one of the key methodologies of evaluating a power system to maintain satisfactory reliability and stability under disruptive and dynamic conditions, across all the demand conditions that are likely to occur. Studies have shown that major disruptions such as the sudden loss of heavy loads or generators in constrained power systems lead to considerable voltage decline and, in general, to voltage instability and collapse [24-27]. Such situations have led to major blackouts, as seen in the August 2003 outage that affected Canada and the USA; the September 2003 blackout in Sweden and Denmark; the July 2004 event in southern Greece [28]; and similar incidents in other countries, as noted in [25, 29-31].

The present-day power system is vulnerable to several types of disturbances, which may arise from changes in control actions, fluctuations in load, short circuits on transmission lines, or the outage of a major generator [32-33]. The power grid assessment is one such strategy towards stabilizing the system [25, 34]. When losses occur, the system must be brought back to an N-1 safe state within 15-20 minutes to limit the potential for further losses, and the

Peter Obinna Mbamaluikem, Oluwaseun Ibrahim Adebisi, and Kamoli Akinwale Amusa, "Survey On Contemporary Methods For Power System Voltage Stability Evaluation," *International Journal of Multidisciplinary Research and Publications (IJMRAP)*, Volume 8, Issue 8, pp. 80-96, 2026.

capability to learn, adapt, and act sensibly [35-39]. Moreover, the core objective of the utility operator is to ensure the continuous delivery of stable and reliable electric power to consumers. However, instability in voltage can hinder the achievement of this goal and reduce the performance of the power system grid. To mitigate such challenges, voltage stability evaluation is performed to maintain the power system reliability and resilience.

The research paper attempts to critically review modern techniques for assessing voltage stability in high-voltage transmission networks. This paper, in Section 2, discusses the background and theoretical foundations relating to power system stability. In Section 3, a detailed classification and characterization of PSS is given. Section 4 contrasts various approaches embraced towards voltage stability research. Section 5 gives a review of the available literature on the power system stability analysis. Section 6 discusses major findings of the study, while Section 7 provides the study's conclusion and outlines directions for further investigation.

II. DYNAMICS OF POWER SYSTEM STABILITY

The development of transmission infrastructure leads to the increased complexity of the networks of the power systems, making it harder to evaluate and maintain the voltage stability [40]. This growth in the network causes voltage sag and diminution of the damping torque of the system, which may result in voltage collapse and blackouts. Therefore, for the assurance and persistence of power system stability, the network layout should be engineered to possess sufficient stability margin, with automatic regulation of generator excitation, and with automatic counter-emergency equipment [41].

The power system stability (PSS) describes the power system's ability to restore its operating condition to its original form, or to a form that is substantially nearly the same as its original form, after a disturbance [42]. Therefore, an interconnected power network is said to be resilient when it can recover after suffering a disturbance and still maintain its stability, where its system variables are within its acceptable operating ranges after the event [43]. It simply implies that the system should be in a position to dampen the oscillations as fast as possible, so that the entire system can return to its original position or to a new steady state point of operation within a finite period. Moreover, the nature of the disturbance, as well as the initial operating state of the system dictate whether an interconnected power system can recover its original stability state or attain a new one due to the disturbance. These aberrations can be termed as the system conditions, such as the variations of loads, short-circuit disturbances, voltage variations, and the loss of transmission lines. Consequently, each power system has a set of critical limit values that could be singled out in relation to those quantities that have an impact on the stability limit.

III. CATEGORIES OF POWER SYSTEM STABILITY

To better address PSS issues and support effective research, the 2004 study by "CIGRE and the IEEE Power Systems Dynamic Performance Committee" categorized PSS based on the nature of disturbances (disturbance magnitude and duration) into frequency, rotor angle, and voltage stability [27, 40]. The dynamics of synchronous generators formed the basis of this classification. This is because the method of electrical power generation was solely by the synchronous generators [41-42]. Nonetheless, integration of the renewable energy source into the existing grid by the use of power electronic conversion technologies, whose dynamic nature varies immensely as compared to the conventional synchronous generators influence the stability of power systems. Therefore, in 2020, with the development in the sector of renewable power (solar and wind power), IEEE, through a technical report named Stability Definitions and Characterization of Dynamic Behaviour in Systems with High Penetration of Power Electronic Interfaced Technologies, identified two additional types of stability, namely "converter-driven stability and resonance stability" [41, 43]. Thus, the enhanced power system stability classification, as presented in CIGRE IEEE 2004 and IEEE 2020, is illustrated in Fig. 1.

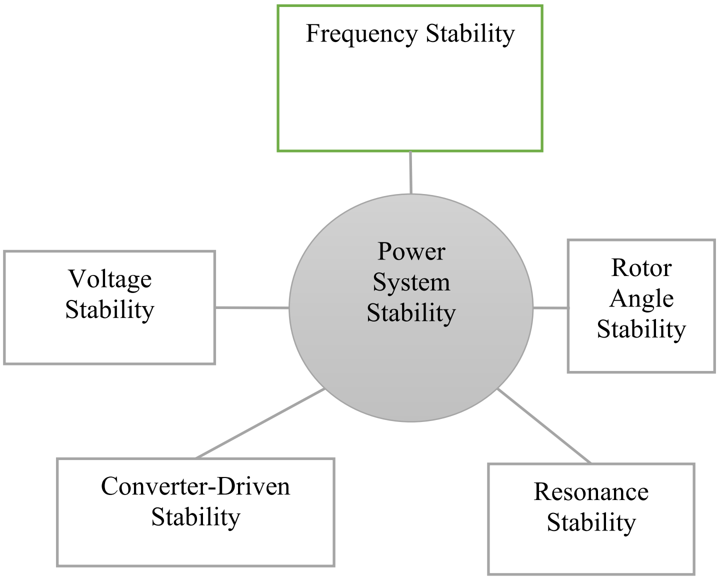

Figure 1: IEEE Extended Classification of Power Stability in 2020 [30-31]

A. *Rotor Angle Stability*

The rotor angle is a key factor in alternator load sharing, since the active power supplied by synchronous generators in a power system depends directly on their rotor angle [1]. Thus, a rise in the prime mover speed will cause forward movement of the rotor angle to a new position with the stator magnetic field, while a decline in the prime mover speed will cause a return of the rotor angle to the main (stator) field. This has repercussions on the speed difference and angular displacement between the alternators [2]. This concept is the reason that synchronous devices in a power system can reestablish synchronism after it has been disrupted, normally 3-5 seconds in the case of transient disturbance types and 10-20 seconds in the case of small disturbance types. Instability happens in this regard whenever the angular separation exceeds an acceptable limit, and synchronous generators lose synchronism.

Hence, when an oscillation occurs, the changes in the rotor angular position of the generator constitute two parts of the electromagnetic torque. One component represents the synchronizing torque, characterized by rotor angle deviations that are in phase alignment with the torque response, while the other is a restraining torque component with the speed deviation in phase with it [3]. These components of the torque should be suitably present if the system is to be maintained in synchronism after a contingency [4]. Absence of the former will result in a type of stability referred to as aperiodic (non-oscillatory) instability. Where the latter is absent, it will result in periodic or oscillatory instability. Angle stability may also be subdivided into two main categories: transient disturbance stability and small-disturbance stability [48-49]

B. Frequency Stability

This stability describes a state in which the amplitude of the power system frequency remains unchanged (constant) at a normal value following the appearance of a disturbance [50-51]. Therefore, an interrelated power system can, as a result, remain in frequency balance between generation and load, following a system contingency [52-54]. Under the condition of abnormality in the system frequency relative to the set reference, the ensuing variations to active power balance may result in generation deficits and/or load shedding. Frequency control, therefore, plays an important role in the operation of a power system because it indicates real-life equilibrium between the load demand and the output of the generators. Poor frequency control may jeopardize the stability of the overall system, and it is therefore likely to create large-scale disturbances or blackouts [55-56]. The frequency instability may be classified based on the time responses of the equipment and processes involved with us, usually being short-term and long-term [57-59].

C. Resonance Stabılıty

Resonance occurs whenever a periodic and oscillating exchange of energy takes place. As noted in [60], the oscillations may become more pronounced in the absence of energy dissipation along the energy flow path and appear in the power system network as a rise in current, voltage, or torque values. The instability induced by resonance has been deemed to arise when the parameters observed in the system exceed the set threshold parameters, which indicates that the conditions in that system have transformed to unstable conditions. In [61], resonance stability is divided into torsional resonance and electrical resonance, in which converter-grid interactions are related to the turbine generator shaft dynamics. The resonance may undermine the stability of the power system and, in extreme cases, cause the equipment to be damaged or fail. These oscillations can be either poorly/undamped, growing, or negatively damped. Common contributing factors include interactions with devices such as HVDC transmission systems, static synchronous compensators (STATCOMs), power system stabilizers, and static VAR compensators (SVCs) [62]. The electrical resonance is purely electrical and therefore may be termed the "Induction Generator Effect." Moreover, it has been confirmed that the electrical resonance has never occurred in conventional generators previously [62].

However, the integration of doubly fed induction generator (DFIG) technology within Type-3 adjustable-speed wind turbine systems can be susceptible to a self-excitation form of induction generator sub-synchronous resonance. In such configurations, grid integration of a DFIG in the presence of series-compensated transmission lines may induce resonant interactions between the generator and the compensator, potentially leading to equipment damage, reduced operational reliability, or even system-wide instability.

D. Converter-Drıven Stabılıty

In contemporary power systems, converter-based generation within the power system is increasing as different countries transform to renewable sources of energy to solve their energy issues [63-64]. Such vogue renewable energy generating stations (solar and wind power stations) use power electronic converters to deliver generated power from the source to the grid [5]. In practical applications, the operational characteristics and dynamic behavior of this emerging technology differ significantly from those of conventional synchronous generators [6]. Consequently, it brings a new type of instability that differs significantly from the usual and traditional operation of machines in synchronous modes. This instability is expressed in terms of large-scale system behaviour due to slow dynamic processes between the electromechanical dynamics of synchronous generators and their controllers, and those of power-electronic-based devices, which are rapid, switching, control-oriented. The interactions can lead to an undesirable oscillatory mode, which may bring about loss of stability and reliability of the entire system [7]. These generating units typically implement a phase-locked loop (PLL) mechanism to achieve synchronization between their operating frequency and that of the grid.

In most cases, such generation units employ a phase-locked loop (PLL) to synchronize the inverter output frequency with that of the grid. The PLL may also serve as a reference signal for synchronizing the generator and the inverted frequency. The PLL and inner current loops are capable of coupling with the power system and can be the cause of oscillations in a large range of frequencies [68-69]. Converter-dominated oscillations are of two types, depending on the interaction rate with the power system, as slow interactions and fast interactions. [8]. Rapid oscillations in converter-dominated systems emerge from dynamic interactions among power-electronic elements, including synchronous generator control systems, HVDC transmission links, FACTS devices, and the wider transmission infrastructure [70-71]. Converter-driven instability, like voltage stability issues, can arise from a weak system or from operating close to the maximum allowable power transfer between the converter and the grid. The distinction is that instability driven by the converter is related to the controls of the electronic converter, while voltage instability is load-driven [9].

E. Voltage Stability

This classification of stability highlights voltage as a key system variable that tends to fluctuate during disturbances across the interconnected power system network. Accordingly, the voltage stability (VS) is the ability of a power system to maintain a balance between power production and consumption, or to maintain the equilibrium between power supply and consumption, and also to make sure that the bus voltages stay within permissible limits, even under disturbance or faults [72-77]. Consequently, the system is considered voltage stable when the bus voltages remain steady after the system has been perturbed from a given initial operating state, and loses stability when any bus within the network experiences a continuous decrease in voltage magnitude [40, 78-79].

Voltage instability is one of the most important phenomena that affects the reliable operation of electric power systems. When the network is unable to sustain acceptable voltage levels, voltage instability is said to have occurred. This phenomenon is usually triggered by inadequate reactive power support, the dynamic nature of electrical loads, and voltage drops associated with active and reactive power that passes through the inductive elements of the transmission system, challenges in generator capability, and the overall topology or configuration of the power network [43, 80]. Furthermore, system faults may induce voltage deviations beyond acceptable limits at certain buses within the power network when the power system fails to deliver the reactive power needed to support the prevailing demand. Such conditions can lead to significant voltage deviations, which may trigger protective devices and result in the disconnection of large sections of the network [42, 81]. VS in a power system is primarily influenced by the availability and allocation of reactive power at individual buses, as well as by voltage drops along transmission lines. Moreover, monitoring how a voltage magnitude responds, particularly how it increases when a reactive power is injected at a critical bus, is the most common method of checking the voltage stability. However, should the bus voltage continue to drop instead of rising, the system may be approaching a state of voltage instability, potentially culminating in voltage collapse [42, 11].

IV. VOLTAGE STABILITY CLASSIFICATION

Voltage Stability (VS) is typically classified according to the severity and duration of disturbances, as small- or large-disturbance types and short- or long-term time frame. Large-disturbance voltage stability is the ability of a power system to sustain and restore acceptable voltage levels after a serious disturbance caused by a short-circuit fault or a generator outage [10]. Large-disturbance stability Studies generally focus on the dynamics on a time scale of a few seconds to a few minutes.

Furthermore, small-disturbance voltage stability characterizes the power system’s ability to preserve or re-establish acceptable voltage magnitudes at all buses after slight variations in generation or load. These events typically fall within a time frame of 0 to 10 seconds. Furthermore, all voltage stability disturbances can further be subclassified based on time limit, analysis approach, and subtypes, as presented in Table 1 [11].

A. Short-Term Voltage Stability

Short-term voltage stability (STVS) is the power system’s ability to maintain or quickly restore acceptable bus voltages following disturbances that involve fast-acting components. These include the rapid dynamic behavior of induction motor loads, electronically switched capacitive devices (capacitor banks or STATCOMs), HVDC interconnectors, automatic voltage regulators, turbine governors, and inverter-based resources [12]. The time scale of STVS is necessarily within the range of a few seconds, relative to the phenomena experienced in rotor angle stability and slow-growing instabilities in converter dynamics. The precise assessment of the STVS requires intensive dynamic load modeling, as it is about fast system response following disruptions. It has been established that a primary contributor to STV instability is the induction motor's lock following a significant disturbance, typically resulting from an imbalance between mechanical and electromagnetic torque, or a delay in clearing the fault that prevents the system from returning to a stable state [13]. Of particular concern are short-circuit faults that occur near the load, which demand immediate attention [14]. Motors experiencing a locked-rotor condition can be isolated through undervoltage protection or allowed to remain in operation until tripped by thermal overcurrent protection. However, when relying on the latter, the voltage is depressed over long periods, and that may lead to a cascading effect, causing adjacent motors to experience similar conditions

B. Long-Term Voltage Stability

The underlying causes of LTV (long-term voltage) instability generally arise from a loss of equilibrium over extended periods, primarily due to the dynamic behaviour of the load as it tries to revise the power absorbed over the maximum power transfer threshold [15]. This type of instability can also occur whenever a corrective measure uses the post-disturbance equilibrium point as a pivot, but with a time lag, to prevent convergence to the equilibrium. It typically shows up as a gradual, step-by-step decrease in voltage at specific buses within the network, reflecting how voltage levels drop in stages rather than all at once [88-89]. Moreover, performing the long-term simulation requires the system's dynamic performance analysis since the time of interest under observation is several minutes.

Such stability is not normally controlled by a fault lying underneath, but rather by the consequent loss of transmission/generation equipment when the fault is cleared. The stability of LTV is generally measured on the basis of the stability margin, which measures the degree of the load level that can be raised under given conditions to a point where the power limit of the system is reached.

To focus on this, the direction of stress, the manner in which the load will be increased, and the manner in which the generation supports the increase, need to be defined. The methods of linear and nonlinear analysis are normally employed to give a more comprehensive picture of the system behaviour [87-88]. Linear analysis measures the stability of a certain working condition by examining the eigenvalues of the Jacobian matrix of the system, the highest point of power transfer, and sensitivity information. Nonlinear models, conversely, can model more complex behaviour of a system, such as operational limits, deadbands, discrete tap changer behaviour, and constant and variable time delays.

Disruption of any type, including small deviations and significant disturbances, ought to be considered when examining the LTV stability of transmission lines. This is a result of the capacitive nature of extra-high and high-voltage lines that have a lower load due to capacitance surge-induced loading, in addition to shunt capacitors and filters at each HVDC terminal. Also, under-excitation limiters do not allow synchronous compensators and generators to absorb too much reactive power. The system, therefore, has difficulties in sustaining stability when the load falls below a definite minimum, and the instability is compounded.

TABLE 1: Voltage stability classification

Classification Type	Subtype	Time Frame	Method of analysis	Cause
Time-Based	Short-Term	Seconds to 1 minute	Quasi Steady State (QSS) time domain simulation, Lyapunov exponent, decision tree (DT)	Fast-acting load dynamics like motor stalling, generator excitation systems, HVDC converters, FACTS devices, e.g., SVCs, STATCOMs
	Long-Term	Several minutes to Tens of Minutes	Eigenvalues of an appropriate Jacobian matrix	Slow-acting load dynamics, tap-changing transformers, and generator AVR response
Magnitude-Based	Large-Disturbance	Varies	Time-domain simulations, dynamic models of loads and controls	Major faults, outages
	Small-Disturbance	Varies	Eigenvalue analysis, sensitivity methods	Minor load changes
System Behaviour-Based	Static Voltage Stability	Steady-state	P-V and Q-V curves	Planning and operational studies
	Dynamic voltage stability	Time-varying	Dynamic simulation tools, time-domain analysis	Real-time operation, control, and protection responses

C. *Sources Of Voltage Instability In Power Systems*

Reactive power requirements, generation capacity, supply-demand imbalances, and transmission line voltage drops determine to a great degree the stability of voltage in a power system. Figure 3 shows different parameters influencing power system network voltage stability [16].

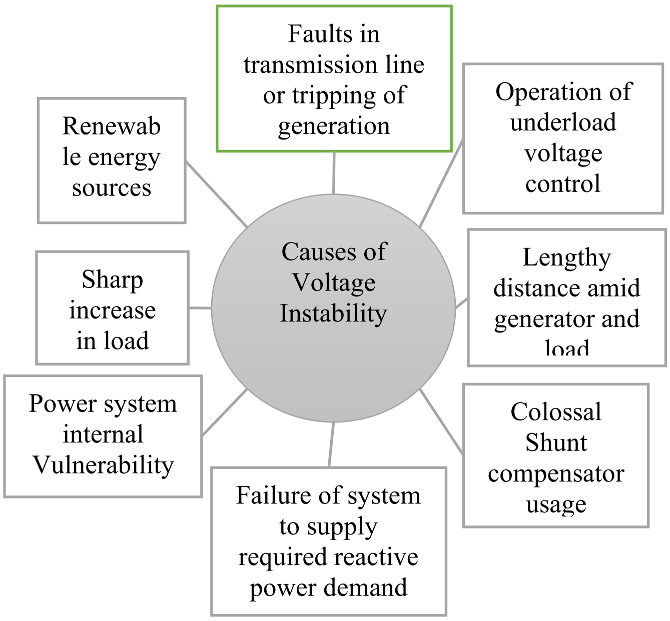

Figure 3: Causes of Voltage Instability in Electrical Grid [30-31]

V. Methods of Voltage Stability Assessment

Voltage stability analysis (VSA) is crucial in detecting possible problems at initial stages and taking preventive actions to prevent any inconveniences that may result in voltage collapse, and consequently, maintaining reliability in the operation of the electrical power system [15]. Its outcome helps in detecting weak, uncontrollable, or unstable zones of the power system, which may potentially threaten or cause problems in future load growth, thus preventing system instability [72, 89]. In addition, VSA is generally categorized into three main types: static, dynamic, and measurement-based approaches [17]. The categorization is shown in Figure 4.

A. *Methods For Assessing Static Voltage Stability*

Methods for static voltage stability analysis (SVSA) include power flow analysis, P-V curve plotting, P-Q sensitivity analysis, Q-V modal analysis, minimum singular value, continuation power flow, impedance-based method, and voltage stability pointer [91-94], as shown in Figure 4. To analyze a power system in its static nature, information such as closeness to voltage collapse, maximum loadability, voltage stability margin, and the limits within which voltage can remain stable is obtained [92-93].

Power Flow Analysis

The power flow analysis (PFA) is a popular technique to determine the magnitude of bus voltage and phase angle at each bus in a power system under normal working conditions [21, 74, 91]. It plays a significant role in determining the power flows through transmission lines, estimating the power losses, and other important operating aspects of the system. It also assists engineers to maximize the performance of the system, and consumers are provided with a quality supply of power. It is applied in power system studies to assess voltage profiles across different operating scenarios. Hence, depending on the levels of loads and generations, this method calculates the system voltages at each bus of the system in a manner that accurately identifies possible voltage stability issues (low-voltage buses or reactive power unbalances). Real and reactive power equations are formulated using the N-bus single-line diagram depicted in Figure 5 as a basis.

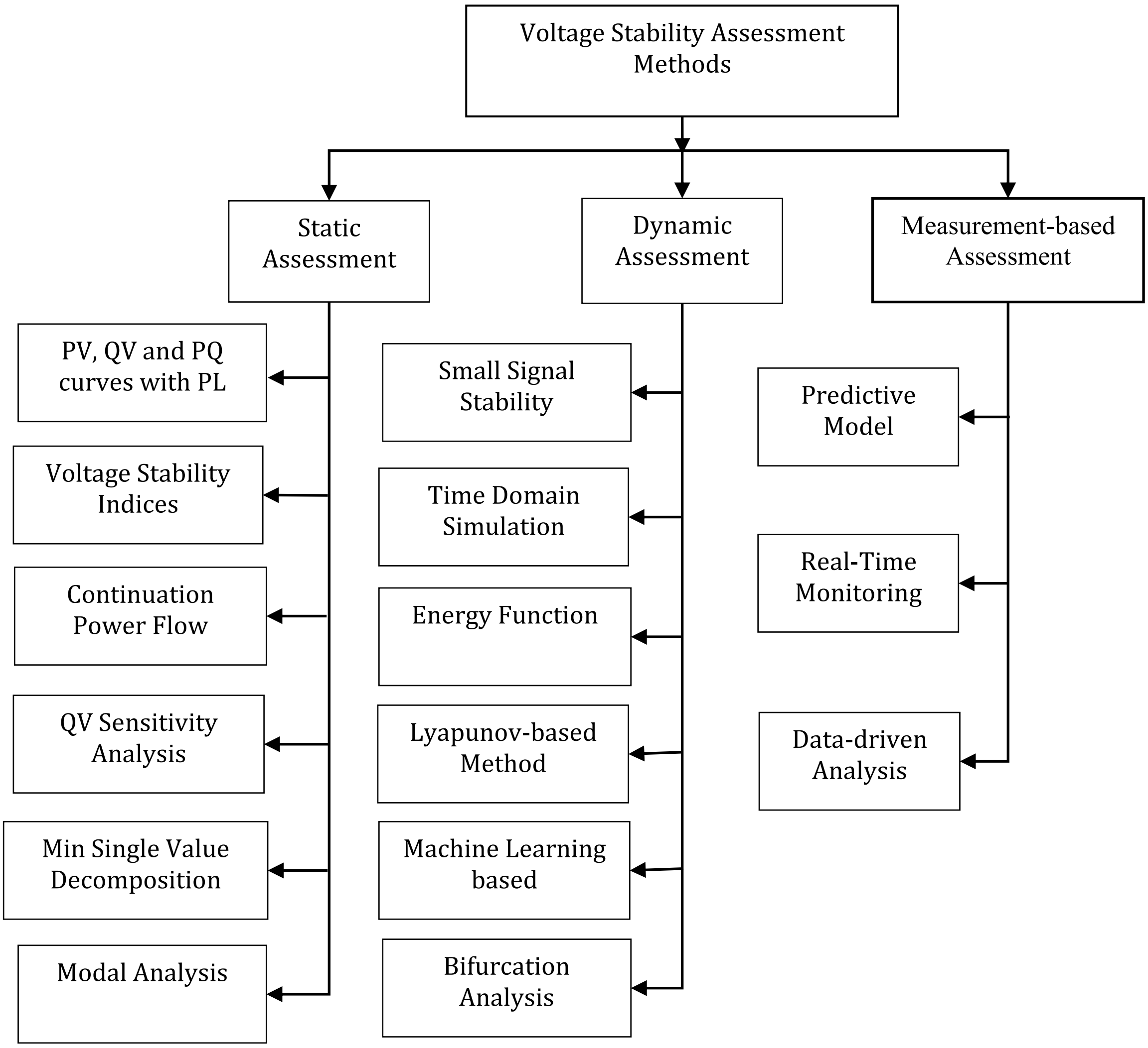

Figure 4: Methods of Voltage Stability Assessment [25, 31]

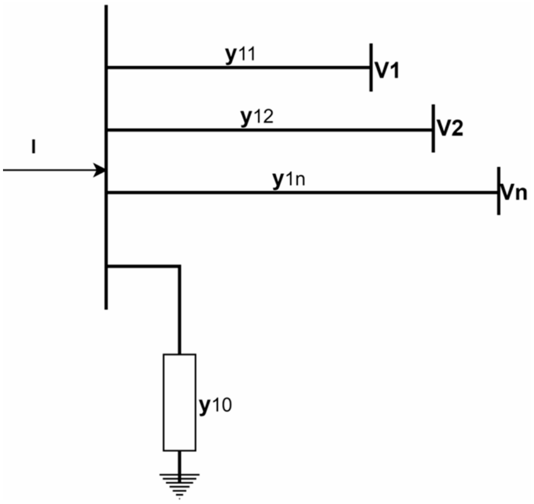

Figure 5: i-th bus line diagram for the power system network [18].

Using KCL (Kirchhoff's current law), the net injected current, I_i at the i-th bus is expressed as:

$$I_i = I_{i0} + I_{i1} + I_{i2} + \cdots + I_{in} \qquad (1)$$

But,

$$\left.\begin{aligned} I_i &= \frac{V_i}{Z_{ij}} = V_i Y_{ij} \\ \text{since}, Y_{ij} &= \frac{1}{Z_{ij}} \end{aligned}\right\} \qquad (2)$$

Therefore,

$$I_i = y_{i0}V_i + y_{i1}(V_i - V_1) + y_{i2}(V_i - V_2) + \cdots + y_{in}(V_i - V_n) \qquad (3)$$

$$I_i = Y_{ii}V_i + Y_{i1}V_1 + Y_{i2}V_2 + \cdots + Y_{in}V_n \qquad (4)$$

In summary,

$$I_i = \sum_{\substack{k=1 \\ k \neq i}}^{n} Y_{ik}V_k \qquad (5)$$

The injected i-th bus real and reactive power is:

$$P_i - jQ_i = V_i^* I_i \qquad (6)$$

Substitute equation (5) into (6)

$$P_i - jQ_i = V_i^* \Big(\sum_{\substack{k=1 \\ k \neq i}}^{n} Y_{ik}V_k\Big) \qquad (7)$$

Considering the angles, let:

$$\left.\begin{aligned} Y_{ii} &= |Y_{ii}|\angle\theta_{ii} \\ V_i &= |V_i|\angle\delta_i \\ Y_{ik} &= |Y_{ik}|\angle\theta_{ik} \\ V_k &= |V_k|\angle\delta_k \\ V_i^* &= |V_i|\angle-\delta_i \end{aligned}\right\} \tag{8}$$

Substituting these values into equation (7) yields:

$$P_i - jQ_i = |V_i|\angle-\delta_i\left(\sum_{\substack{k=1 \\ k\neq i}}^{n}|Y_{ik}|\angle\theta_{ik}|V_k|\angle\delta_k\right) \tag{9}$$

$$P_i - jQ_i = \sum_{\substack{k=1 \\ k\neq i}}^{n}|Y_{ik}||V_k||V_i|\angle(\theta_{ik}+\delta_k-\delta_i) \tag{10}$$

Transforming the RHS of (10) into rectangular form results in:

$$P_i - jQ_i = \sum_{\substack{k=1 \\ k\neq i}}^{n}|Y_{ik}||V_k||V_i|\cos(\theta_{ik}+\delta_k-\delta_i) + j\sum_{\substack{k=1 \\ k\neq i}}^{n}|Y_{ik}||V_k||V_i|\sin(\theta_{ik}+\delta_k-\delta_i) \tag{11}$$

Separating real and imaginary parts gives:

$$P_i = \sum_{\substack{k=1 \\ k\neq i}}^{n}|Y_{ik}||V_k||V_i|\cos(\theta_{ik}+\delta_k-\delta_i) \tag{12}$$

$$-jQ_i = j\sum_{\substack{k=1 \\ k\neq i}}^{n}|Y_{ik}||V_k||V_i|\sin(\theta_{ik}+\delta_k-\delta_i) \tag{13}$$

$$Q_i = -\sum_{\substack{k=1 \\ k\neq i}}^{n}|Y_{ik}||V_k||V_i|\sin(\theta_{ik}+\delta_k-\delta_i) \tag{14}$$

Expanding equations (12) and (14) using the Taylor Series, we obtain:

$$\begin{bmatrix} \Delta P_2^{(j)} \\ \vdots \\ \Delta P_n^{(j)} \\ - \\ \Delta Q_2^{(j)} \\ \vdots \\ \Delta Q_n^{(j)} \end{bmatrix} = \begin{bmatrix} \frac{\partial P_2^{(j)}}{\partial \delta_2} & \cdots & \frac{\partial P_2^{(j)}}{\partial \delta_n} & \frac{\partial P_2^{(j)}}{\partial V_2} & \cdots & \frac{\partial P_2^{(j)}}{\partial V_n} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ \frac{\partial P_n^{(j)}}{\partial \delta_2} & \cdots & \frac{\partial P_n^{(j)}}{\partial \delta_n} & \frac{\partial P_n^{(j)}}{\partial V_2} & \cdots & \frac{\partial P_n^{(j)}}{\partial V_n} \\ - & - & - & - & - & - \\ \frac{\partial Q_2^{(j)}}{\partial \delta_2} & \cdots & \frac{\partial Q_2^{(j)}}{\partial \delta_n} & \frac{\partial Q_2^{(j)}}{\partial V_2} & \cdots & \frac{\partial Q_2^{(j)}}{\partial V_n} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ \frac{\partial Q_n^{(j)}}{\partial \delta_2} & \cdots & \frac{\partial Q_n^{(j)}}{\partial \delta_n} & \frac{\partial Q_n^{(j)}}{\partial V_2} & \cdots & \frac{\partial Q_n^{(j)}}{\partial V_n} \end{bmatrix} \begin{bmatrix} \Delta\delta_2^{(j)} \\ \vdots \\ \Delta\delta_n^{(j)} \\ - \\ \Delta|\Delta V_2^{(j)}| \\ \vdots \\ \Delta|\Delta V_2^{(j)}| \end{bmatrix} \tag{15}$$

Equation (15) can be further simplified to:

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} J_1 & J_2 \\ J_3 & J_4 \end{bmatrix} \begin{bmatrix} \Delta\delta \\ |\Delta V| \end{bmatrix} \tag{16}$$

The differential residual power and bus voltages are estimated to be obtained from $J_1, J_2, J_3\ and\ J_4$, diagonal and off-diagonal components as:

$$\Delta P_i = P_{Gi} - P_{Li} \tag{17}$$

$$\Delta Q_i = Q_{Gi} - Q_{Li} \tag{18}$$

$$\delta_i = \delta_{Gi} + \Delta\delta_{Li} \tag{19}$$

$$|V_n| = |V_{il}| + \Delta|V_{ij}| \tag{20}$$

Continuation Power Flow

The continuation power flow (CPF) method is an extension of the classical load flow study, used to trace the system's operating points as loads or generation vary. It also rewrites the classical load flow equations in such a way that it is numerically stable at the singularity point where the P-V curve folds to allow both the upper and lower parts of the curve to be computed correctly [95-96]. The method yields information about the stability of voltages in a parameter known as the loading margin (LM). LM is the limit of power system load that can be added to the system without affecting its stability. It helps in voltage stability examination by monitoring the variations in voltage as the load is added [97-98]. It is mostly useful for simulating voltage collapse conditions and identifying the load critical levels at which voltage instability will emerge.

It calculates system response at different loading conditions and allows the system to adjust continuously till it reaches a voltage collapse point. The process begins with predicting the solution of the next value of the continuation parameter. Then, this prediction is improved using the actual load flow results. During the correction step, a parameter denoted as a continuation parameter is set, and the original linear prediction is modified to match the non-linear equations of the prediction [76, 99].

Contingency Analysis

The contingency analysis (CA) measures the effects of various system contingencies on the voltage stability. The contingencies include a generator outage, loss of transmission line, or a transformer [19]. Consequently, it estimates the impact of the outage of a single unit in the power system on the overall performance of the power system, providing valid information that enables the operator to prevent system failure and blackouts. Thus, the task is also considered an N-1 contingency analysis [19]. CA assists in recognizing vital systems and coming up with plans to ensure the stability and reliability of the grid in the event of contingencies. In the N-1 contingency analysis, the flows of the branches are compared to the thermal limits of the transmission lines and the loading limits of the transformers to ensure that the Bus voltages do not exceed allowable values in comparison to the nominal values. The simulation of different failure modes assists in making the operators aware of the critical parts of the network, which tend to become unstable in terms of voltage during certain disturbances. In determining the stability issues, the CA solves a nonlinear power flow system of equations using Newton–Raphson's iterative technique. The operating limits of the network buses and branches are then checked after solving the power-flow equations of each contingency scenario to ensure that their operating limits do not exceed acceptable limits.

V-Q Sensitivity Analysis

The sensitivity analysis examines how the system's voltage profile responds to variations in parameters such as load, generation, and network configuration. It helps in determining the vulnerable areas in the network and also the parameters most likely to lead to voltage instability. More so, voltage sensitivity to changes in reactive power injection can help identify buses or areas where reactive power support should be installed. The Q-V sensitivity analysis can be formulated using the power flow equations together with the system's Jacobian matrix, as shown in (21) - (26):

$$\begin{bmatrix}\Delta P\\ \Delta Q\end{bmatrix} = [J_1]\begin{bmatrix}\Delta\delta\\ |\Delta V|\end{bmatrix} \qquad (21)$$

Where:

ΔQ is the mismatch in reactive power; ΔP denotes changes in real power; $\Delta\delta$ represents incremental variations in the bus voltage angle; $|\Delta V|$ is the incremental deviation in the bus voltage magnitude.

Jacobian matrix, $J = \begin{bmatrix}J_{P\delta} & J_{PV}\\ J_{Q\delta} & J_{QV}\end{bmatrix}\begin{bmatrix}\Delta\delta\\ |\Delta V|\end{bmatrix}$ (22)

The QV sensitivity of the i^{th} load bus is the i^{th} diagonal elements of the matrix, $[J_{QV}]$.

Assuming $\Delta P = 0$. The reduced Jacobian equation becomes:

$$\begin{bmatrix}0\\ \Delta Q\end{bmatrix} = [J_R]\begin{bmatrix}\Delta\delta\\ |\Delta V|\end{bmatrix}, \qquad (23)$$

$$\Delta Q = J_R\Delta V \qquad (24)$$

$$J_R = [J_{QV} - J_{Q\delta}J_{P\delta}{}^{-1}J_{PV}] \qquad (25)$$

In (25), J_R defines the relationship between reactive power injected at the buses and the corresponding voltage magnitudes. From (24):

$$\Delta V = J_R{}^{-1}\Delta Q \qquad (26)$$

$J_R{}^{-1}$ is the inverse of the reduced V-Q Jacobian matrix. The diagonal entries of this matrix correspond to the voltage sensitivity at each bus. Furthermore, the slope of the V-Q curve at a particular operating point reflects the voltage-reactive power sensitivity: a positive slope denotes a stable operating condition, while a negative slope indicates potential instability. Generally, Lower positive sensitivity values indicate a more stable system, whereas higher negative sensitivity values correspond to increased instability [25, 101].

Q-V Modal Analysis

It is useful in realizing how various forms of voltage and power affect the overall stability of the system. It allows a closer look at the behaviour that supports the phenomenon of voltage instability, such as the capacity of certain modes of the system to either cause or alleviate instability as a result of minimal disturbances [20]. Under the solution in the focus of the Modal analysis, the linearized system equations are used to find the eigenvectors, which are the natural modes of oscillation of the system. The two modes will be Current and voltage oscillation pattern. The damping of every mode is measured by looking at its eigenvalues; the real part of the eigenvalues gives the damping, and the imaginary part gives the frequency of oscillation. An eigenvalue of a mode can be close to zero (i.e. lack of damping), which can cause voltage instability [21].

To estimate the power system voltage stability margin and predict the possible occurrence of voltage collapse, modal analysis that is in the form of ΔQ/ΔV is used. The most important in this approach is the determination of the smallest eigenvalues and eigenvectors of the reduced Jacobian that is obtained through load flow analysis. With the solution of the linearized power flow equations expressed as the Jacobian matrix in (18), one can gain the resulting changes in active power (ΔP) and reactive power (ΔQ) as in equation (20):

$$\begin{bmatrix}\Delta P\\ \Delta Q\end{bmatrix} = \begin{bmatrix}J_1 & J_2\\ J_3 & J_4\end{bmatrix}\begin{bmatrix}\Delta\delta\\ |\Delta V|\end{bmatrix} \qquad (27)$$

Considering $\Delta P = 0$, (27) becomes:

$$\begin{bmatrix}0\\ \Delta Q\end{bmatrix} = [J_R]\begin{bmatrix}\Delta\delta\\ |\Delta V|\end{bmatrix} \qquad (28)$$

$$\Delta Q = [J_R]\Delta V \qquad (29)$$

Dividing both sides by $[J_R]$., yields:

$$\Delta V = J_R^{-1}\Delta Q \qquad (30)$$

While,

$$J_R^{-1} = \xi\,\eta\Lambda^{-1} \qquad (31)$$

ξ is the right eigenvector matrix of J_R; η denotes the left eigenvector matrix of J_R; and Δ represents the diagonal eigenvalue matrix of J_R.

Substituting (31) into (30), gives:

$$\Delta V = \xi\,\eta\Lambda^{-1}\Delta Q \qquad (32)$$

In a more general term,

$$\Delta V = \sum_i \frac{\xi_i\eta_i}{\lambda_i}\Delta Q, \qquad (33)$$

λ_i is the i^{th} eigenvalue, ξ_i symbolizes the i^{th} column right eigenvector, and η_i represents the i^{th} row left eigenvector of the reduced Jacobian matrix.

Every eigenvalue, as well as its right and left eigenvectors, specifies one of the modes of the V-Q response. Assuming $\xi\,\eta = I$, and $\xi^{-1} = \eta$. Then, (32) can be expressed as:

$$\eta\Delta V = \eta\Lambda^{-1}\Delta Q \qquad (34)$$

Therefore,

$$v = \Lambda^{-1}q \qquad (35)$$

v signifies the vector of modal voltage variations = ηΔV.

q symbolizes the vector of modal reactive power variations = ηΔQ.

Moreover, the i^{th} Modal voltage variation is given by:

$$v_i = \frac{1}{\lambda_i}q_i \qquad (36)$$

Based on (36), we can infer that a larger eigenvalue, λ_i corresponds to a greater deviation of the associated modal voltage in response to changes in reactive power. Smaller eigenvalues, on the other hand, are more volatile; that is, a small change in q_i may cause a large variance in the modal voltage. In this case, the eigenvalue is 0, meaning that the modal voltage is unstable and may eventually collapse. This failure points to the fact that this system is straining and failing to hold the voltage steady. At this point, one little difference in reactive power can produce large uncontrolled changes in voltage, which means that the system is approaching the point of voltage collapse.

Thus, when one or more of these eigenvalues hit zero, the voltage stability boundary is achieved, and this means that one or more modal voltages have collapsed. The system is considered voltage unstable in case one of the eigenvalues is negative. Generally, the size of an eigenvalue provides an approximate determination of the closeness of the system to the instability of voltages [42, 101]. It is necessary to point out that a reduction in the size of eigenvalues indicates that the system is homogeneously close to a critical or unstable state. To get a more thorough and precise analysis of the voltage stability, one may want to analyze at least 5 to 10 smallest eigenvalues. These modes can tend to be connected with other parts of the power system and demonstrate localized reasons that stability may be a problem. Notably, the mode of the eigenvalue of the lowest value is not necessarily critical, particularly when the system gets more stressed. A bigger analysis offers one a better view of areas that may be vulnerable throughout the network.

Participation Factor Analysis

Participation factor (PF) analysis calculates each generator or load contribution to a specific mode of oscillation or eigenmode, i.e., it informs us about the strength of participation of a specific bus or generator in a particular mode [20]. Moreover, the analysis of power flow can be used to identify the particular components in the system that is the most hazardous to voltage stability. Once the eigenvalues have been used to identify system modes (i.e. once the modal analysis has been completed), PFs are computed on all the modes to find out the most probable relationship or bus that will be jeopardized in the system. The inherent approach aids in establishing the generators or loads that have most importance to voltage stability, thus informing the addition of reactive power support or any other remedial measures. The outage or leave of a generator or bus that highly participates may also undermine the voltage stability within the system. In mode *i*, the participation factor corresponding to bus *k* is defined by the equation below

$$P_{ki} = \xi_{ik}\eta_{ki} \tag{37}$$

It should be emphasized that a bus approaches instability in the context of a specific mode, *i* when the bus is large. Also, for the branch, the relative contribution of branch *j* is expressed by the equation:

$$P_{ji} = \frac{\Delta Q_{loss\ for\ branch\ j}}{max\,\Delta Q_{loss\ for\ branches}} \tag{38}$$

The branches with high values of PFs define the branches that absorb the maximum reactive power in a system with minimum change in reactive load. The insights gained from branch participation factors can be used to select contingencies for emergency studies and to identify ways to reinforce transmission lines. This helps redistribute power flow, reducing the load on heavily stressed lines.

Voltage Stability Indices

Today, voltage stability indices (VSIs) are increasingly used as effective tools to assess how close a power system is to voltage collapse. Comparison with the traditional methods, including PV and QV curves or QV sensitivity analysis, VSIs provide greater and more practical details about the system condition [22, 23]. They help identify critical lines, weak nodes, and loading conditions not easily detected by conventional methods [24]. In Particular, VSIs are useful in measuring the voltage stability at various positions along the system. This enables power system operators and planners to thoroughly evaluate the system's current performance and make well-informed decisions concerning potential risks [27, 104].

Furthermore, VSIs are valuable for identifying the most effective placement and sizing of DG units and FACTS devices in the power system [29, 105]. These insights will support more effective system planning and contribute to greater overall resilience and stability of the power network [25]. VSIs may be applied in three main modes, and each of them serves a different purpose in analysing the power system and its performance [26]. In Online Mode, system operators can monitor VSI values in real time while the system is running. This immediate access allows them to respond promptly and maintain or restore stability whenever signs of stress arise. Additionally, Offline Mode is primarily used for design and planning purposes. In this approach, system designers and engineers analyze VSI values to assess the overall stability of the power system across varying operating conditions, supporting planning and future improvements.

The Real-Time Mode uses Phasor Measurement Units (PMUs) to acquire and transmit real-time data to the system. The devices enable highly accurate and time-synchronous measurement of voltage and current, which is used to provide dynamically responsive measurement of stability.

Typical ranges of VSI values are in the [0,1] range, where values nearer to 0 are close to the instability range, and values nearer to 1 are in a more stable state. From a broad perspective, VSIs can be grouped into two categories: those methods that rely on the Jacobian matrix and involve the mathematical modeling of the system's dynamics, and system-based indices that focus on overall measures of system performance and behaviour. The integration of these tools provides a comprehensive framework for evaluating and maintaining voltage stability in modern power systems [106-107].

As highlighted by the authors in [34], VSIs can additionally be categorized by type, concept, and dependence on line impedance. The word "concept" as used by the authors of the reference [16] denotes theories that govern the construction of the index, whereas "type" denotes an element in a network, i.e., line or bus. Other VSI classifications and the latest VSI indices are presented in [25, 27, 29-30, 34, 87]. Moreover, VSIs based on the Jacobian matrix more accurately compute the absolute voltage stability margin, as they use system parameters to determine the VSM (line power, bus voltages, and admittance matrix), and can be applied to online monitoring [27].

Furthermore, indices based on system variables demand minimal computation time while effectively identifying weak buses or lines with high accuracy. For voltage collapse points and maximum Loadability determination, however, Jacobian-based indices are used as they are very efficient. Even so, owing to the lengthy computation time, it is not preferred for online analysis [107, 109].

B. Methods Of Dynamic Voltage Stability Assessment

The concept of dynamic voltage stability (DVS) is that a power system can maintain tolerable voltages following a perturbation (faults or sudden load changes) and can gradually return to a stable operating state over time. According to the authors in [22], DVS in a power system network is assessed using various methods. These methods typically include modelling and simulation of transient phenomena and control of the system behavior against disturbances. The DVSA employs a range of techniques for simulating and analyzing power system behavior over time, particularly as it approaches voltage instability. These include time-domain simulation, energy function methods, contingency analysis combined with time-domain simulation, numerical techniques such as the Euler techniques, Runge–Kutta methods, explicit or implicit integration techniques, coupled with small-signal stability analysis and bifurcation analysis.

These methods help clarify the sequence of events leading to voltage collapse, enabling engineers and system operators to more accurately forecast, monitor, and respond to potential grid instability, especially during disturbances or sudden shifts in load and generation [20]. DVS indices quantify the closeness of a system to voltage instability under transient disturbances and can be used to estimate the likelihood of voltage collapse or the potential for recovery following such events [28]. A time-dependent index is used in simulations to model the system's dynamic response over time. It monitors voltage profile changes, gauges the system's proximity to instability, and evaluates the system responses under disturbances such as faults, generator outages, or fluctuations in load [29]. These indices can account for factors such as reactive power margin and voltage variations over time.

Direct Method or Energy Function

This technique involves analyzing the system's total energy to identify dynamic voltage stability. Energy functions provide a means of evaluating the stability of equilibrium points, particularly when the system is approaching collapse or nearing the boundary of a stable region. The energy function that applies the Lyapunov direct method depends on the power system state of equilibrium and defines the way the system's energy progresses as it moves away from its stable operating point [30]. The modelling approach assumes that an energy function is used to represent voltage stability, taking into account both the mechanical and electrical energy within the system.

The function allows for the identification of unstable points and can be used to simulate voltage collapse. The method is widely used in both static and dynamic voltage stability modelling and simulation for estimating system voltage collapse by observing whether the energy declines or rises with disturbances [31, 32]. In conjunction with this, the method also considers the stability of the system through an energy-based perspective. To characterize this physical property, the energy equation is then used as a heuristic based on the active power balance equation integrated through the phase angle, and the reactive component is then added to the voltage magnitude [33]. This empirical energy function in the notation of [33], is given as:

$$E = \int_{(\theta^0, U^0)}^{(\theta, U)} [f(\theta, U), g(\theta, U)] \begin{bmatrix} d\theta \\ dU \end{bmatrix} \tag{39}$$

$$E_i = \int_{(\theta_i{}^0, (U_i{}^0)}^{(\theta, U)} [f_i(\theta_i, U_i), g_i(\theta_i, U_i)] \begin{bmatrix} d\theta_i \\ dU_i \end{bmatrix} \tag{40}$$

Where: U_i is the voltage magnitude of bus i; $U_i{}^0$ represents the initial steady-state value; θ_i is the phase angle of bus i; $\theta_i{}^0$ represents the initial steady-state value, and E_i denotes the bus *i* energy

The equation for f_i and θ_i, are given as:

$$f_i(\theta_i, U_i) = P_i - \sum_{j=1}^{n} B_{ij} |U_i||U_j| \sin(\theta_i - \theta_i) - \sum_{j=1}^{n} G_{ij} |U_i||U_j| \sin(\theta_i - \theta_i) \tag{41}$$

$$g_i(\theta_i, U_i) = U_i{}^{-1} \tag{42}$$

Where: G_{ij} denotes the conductance between buses *i* and *j*; B_{ij} is the susceptance of buses *i* and *j*.

Further, the active and reactive power balance without compensation for bus *i* is respectively given as:

$$P_{Li} = U_i \sum_{j=1}^{n} U_j \left[G_{ij} \cos(\theta_i - \theta_j) + B_{ij} \sin(\theta_i - \theta_j)\right] \tag{43}$$

$$Q_{Li} = U_i \sum_{j=1}^{n} U_j \left[G_{ij} \sin(\theta_i - \theta_j) + B_{ij} \cos(\theta_i - \theta_j)\right] \tag{44}$$

Therefore, the energy function relation is given by:

$$\begin{aligned} E_i = \int_{(\theta_i{}^0, (U_i{}^0)}^{\theta_i, U_i} [f_i, g_i] \begin{bmatrix} d\theta_i \\ dU_i \end{bmatrix} &= P_{Li}(\theta_i - \theta_i{}^0) + \frac{1}{2} \sum_{j=1}^{n} B_{ij} |U_i{}^0||U_j{}^0| \cos(\theta_i{}^0 - \theta_j{}^0) \\ &- \frac{1}{2} \sum_{j=1}^{n} B_{ij} |U_i||U_j| \cos(\theta_i{}^0 - \theta_j{}^0) - Q_{Li} \ln\left(\frac{U_i}{U_i{}^0}\right) \\ &- (U_i{}^0)^{-1} \sum_{j=1}^{n} G_{ij} |U_i{}^0||U_j{}^0| \sin(\theta_i{}^0 - \theta_j)(U_i - U_i{}^0) \\ &- \sum_{j=1}^{n} G_{ij} |U_i{}^0||U_j{}^0| \cos(\theta_i{}^0 - \theta_j{}^0)(\theta_i - \theta_i{}^0) \end{aligned} \tag{44}$$

Time-Domain Simulation (Transient Stability Simulation)

A fault, generator trip, or load change is an abrupt change in power systems that alters the system dynamics. The dynamics of the system are then monitored through simulation over time. Therefore, the time-domain simulation is employed to assess the system dynamics by evaluating system response under varying disturbances. It is a process that entails solving the system differential equations with time to track the variation of the voltages and other variables in the system following a perturbation. The dynamic and steady-state behaviour of generators, loads, and transmission lines is determined using the model behaviour based on differential-algebraic equations. This is then succeeded by the voltage behaviour analysis of the system in finding out whether the system is going to pick up and reach a steady point of operation or fail in voltages. This technique gives information on the rate and efficiency of recovery of the system to stable voltage levels following such a disturbance.

Machine Learning and Artificial Intelligence Method

Peter Obinna Mbamaluikem, Oluwaseun Ibrahim Adebisi, and Kamoli Akinwale Amusa, "Survey On Contemporary Methods For Power System Voltage Stability Evaluation," *International Journal of Multidisciplinary Research and Publications (IJMRAP)*, Volume 8, Issue 8, pp. 80-96, 2026.

The latest developments in ML and AI have enabled data-driven approaches for the evaluation of voltage stability [11]. These methods, such as SVM (support vector machines), ANFIS (adaptive neuro-fuzzy inference system), ANN (artificial neural networks), DT (decision trees), and fuzzy logic, are capable of predicting voltage instability using historical data and recognition of patterns that may result in voltage collapse [34]. These technologies can be trained to predict voltage instability events and provide real-time assessments [34, 11]. It has the benefit that, while the remaining methods require significant computational effort and are difficult to deploy in real-time, since they rely on detailed mathematical models of the power system itself, it can be used both online and offline. Therefore, this flexibility makes them particularly useful for real-time voltage stability assessment and explains their growing popularity in modern power system monitoring [35].

Eigenvalue Analysis

The eigenvalue method is among the most commonly used techniques to analyze small-signal stability in conventional power systems [36]. In assessing dynamic stability, the eigenvalues of the entire system Jacobian are evaluated. The system is stable if all eigenvalues have negative real parts, indicating that any oscillations decay over time. Conversely, a positive real part implies insufficient damping or instability in the corresponding mode [37]. Voltage stability studies, however, adopt a related but distinct approach by focusing on the reduced Jacobian matrix, J_R, which captures the sensitivity between bus voltages and reactive power. In this case, the sign convention differs: the system is deemed voltage stable provided that all eigenvalues remain positive [38]. When an eigenvalue is zero, the system is approaching voltage instability; a negative eigenvalue, however, denotes that the system is unstable or that the V-Q relationship at the affected bus is reversed [76]. In practice, engineers focus primarily on the smallest eigenvalues of the J_R because they are the most sensitive indicators of proximity to instability. Tracking these critical values is often sufficient, as the minimum eigenvalue approaches zero, the matrix nears singularity, a strong indication that the system is approaching voltage instability. The mathematical basis for this assessment arises directly from the power flow equations given in (19) – (23), with the Jacobian matrix given as:

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} J_{P\theta} & J_{PV} \\ J_{Q\theta} & J_{QV} \end{bmatrix} \begin{bmatrix} \Delta\theta \\ |\Delta V| \end{bmatrix} \tag{45}$$

Where ΔP represents a small change in real power, and ΔQ represents a small change in reactive power.
Δθ refers to a small variation in the voltage angle, and ΔV represents a small change in voltage magnitude.

Such simplified attention is useful to minimize computational efforts and yet correctly predict possible risks regarding the system's voltage stability.
Considering ΔP = 0, then the simplified Jacobian expression will be:

$$\begin{bmatrix} 0 \\ \Delta Q \end{bmatrix} = [J_R] \begin{bmatrix} \Delta\theta \\ |\Delta V| \end{bmatrix} \tag{46}$$

$$\Delta Q = [J_R]\Delta V$$

Dividing both sides by $[J_R]$. We have: (47)

$$\Delta V = J_R^{-1} . \Delta Q$$

But, (48)

$$J_R^{-1} = \xi \eta \Lambda^{-1} \tag{49}$$

Where: ξ represents the right eigenvector matrix of J_R, η refers to the left eigenvector matrix of J_R, and Δ represents the diagonal eigenvalue matrix of J_R. Substituting (24) into (23), we obtain:

$$\Delta V = \xi \eta \Lambda^{-1} \Delta Q \tag{50}$$

In a more general term,

$$\Delta V = \sum_i \frac{\xi_i \eta_i}{\delta_i} \Delta Q \tag{51}$$

VI. COMPARISON OF STATIC AND DYNAMIC ANALYSES IN VSA

The comparative strengths and limitations of the two principal analytical frameworks (static analysis and dynamic analysis), in power system voltage stability analysis, are presented in Table 3. These two analytical approaches are widely adopted to examine system behaviour under diverse operating conditions, with each framework providing distinct analytical perspectives on the underlying mechanisms of voltage instability.

TABLE 3: The Comparison of Static and Dynamic Analysis in VSA.

Type of Analysis	**Strengths**	**Limitations**
Static analysis	i. Perform static analysis on different system cases. ii. Supports a multi-faceted analysis of the issue. iii. Requires minimal data to perform analysis and is simple to understand. iv. Provides a thorough knowledge of the problem and helps in identifying the root causes. v. Rapid and economical construction	i. Does not account for the dynamic characteristics of voltage stability in the power system. ii. Accuracy is limited. iii. Offers estimated results.
Dynamic analysis	i. Provides an in-depth analysis of certain voltage control conditions. i. Offers a clearer, time-sequenced view of events leading to voltage instability.	i. Takes more time. ii. More costly. iii. Involves big computer power and high model accuracy for complex control. iv. Requires exhaustive information. v. Understanding is tougher.

VII. REVIEW OF VOLTAGE STABILITY ASSESSMENT

Several studies have been conducted on different voltage stability techniques, which have been implemented to support diverse approaches for assessing voltage stability in power system networks.

Djari *et al.* [39] conducted a comparative study of two main approaches, the V-Q Sensitivity Analysis and Modal analysis, to evaluate their effectiveness in identifying weak buses and assessing voltage stability in power systems. This work examines the performance of these methods under two conditions: a situation where the interaction between reactive power (Q) and voltage angle (δ) is weak, and another where this coupling is absent. The result of their analysis shows that the poor Q–δ coupling has no consequence on the results but has improved computational speed.

Nor and Sulaiman [40] apply both the QV and the PV to evaluate voltage stability within power system networks. To carry out these analyses, the MATLAB application software is used. The software provides a computational platform for evaluating system voltage stability under varying operating conditions. It was established that since Q is directly correlated with the magnitude of voltage, it yields a more accurate outcome for participation factors as compared with the magnitude of the voltage.

The application of the modal analytical techniques for estimating voltage stability in complex power systems was introduced in the study by Seedahmed *et al.* [41]. Later on, paper [123-124] applied modal analysis for voltage stability assessment, validating the methods on the IEEE 9-bus, 14-bus, and 30-bus test systems.

In [42], the authors proposed a new stability analysis voltage indicator, the Modified Stability Assessment Index (MSAI). This index was applied to assess power system stability under different operating conditions and to prioritize system contingencies. The approach was validated using the IEEE 30-bus, 57-bus, and 118-bus test systems, with all simulations performed in the MATLAB environment. The primary strengths of MSAI are that it incorporates all the parameters associated with different transmission lines, making it a robust and holistic tool for studying voltage stability.

Ma *et* al [43] presented a review of methodologies for assessing voltage stability in power systems. The paper surveyed various methods for evaluating voltage stability and highlighted prospective directions for future research.

The Wide-Area Measurement voltage stability sensitivity methodology was introduced in [44], building on the L-index analytical approach. In this study, the authors developed simplified sensitivities, namely, the L-index, L'-index, L-Q index, L'-Q index, and L-P index, L'-P index. These simplified indices reduce computational complexity, making the proposed sensitivities adaptable for large-scale power systems and effective for voltage stability analysis.

Palepu *et al.,* [45] integrated PMUs and STATCOMs for online, real-time monitoring and control of voltage stability margins. In their approach, PMUs were used to estimate the voltage stability margin, while STATCOMs provided compensation for reactive power deficiencies at the critical bus where they were deployed.

Derakhshandeh and Golshan [46] presented an innovative approach to compute the loadability margin for online VSA using PMU measurements and the Thevenin equivalent.

Furthermore, an innovative voltage stability assessment algorithm is presented in [47] for forecasting the loadability margin in a power system. According to the authors, the algorithm can accurately estimate the system's voltage stability stiffness using three samples of bus voltage measurements along with the bus outgoing power.

In [48], Djari and Benasla applied modal analysis, V-Q sensitivity, and continuation power flow methods to identify weak buses in the western Algerian power system during a voltage stability assessment. The study further analyzed how weak coupling between reactive power (Q) and voltage angle (δ) influences overall system performance.

Ghimire *et al.* [49] employed modal analysis to determine the voltage-stable regime of the Nepalese power system. The study also introduced a heuristic approach that utilizes modal analysis results to guide the system from an unstable state back to stable operation.

A new index, the Super Voltage Stability Index (SVSI), was proposed in [24] to quantify the voltage stability margin in power systems. Derived from the reactive power loss method, the SVSI also incorporates N-1 contingency scenarios to evaluate voltage sensitivity.

Song *et al.* 133] proposed a new line voltage collapse index for voltage stability studies based on Q-V sensitivity. To evaluate system stability, the authors introduced two measures: the State-In-Mode Participation Factor (SIMPF), which quantifies the contribution of a specific system state to a dominant mode, and the State-In-Mode Sensitivity (SIMS), which assesses the responsiveness of that state to control actions targeting the most vulnerable oscillatory mode.

In a related study, Gupta *et al.* [50] proposed a new line voltage collapse index for assessing voltage stability using Q–V sensitivity analysis. Derived from the ABCD parameters of the system's reactive power equations, the index can forecast voltage collapse under varying loading conditions and N-1 contingency scenarios.

The authors in [22] undertook a detailed comparative study of existing voltage stability assessment techniques, definitions, and criteria for evaluating voltage stability. The study led to a new definition of voltage stability and a strategic plan for future system upgrades aimed at enhancing stability and performance.

Werkie *et al.* [25] investigated voltage stability phenomena through line- and bus-level stability indices, as well as offline and real-time evaluations that account for load uncertainty, renewable energy integration, and network topology variations. Their work also presented a systematic review covering key aspects of mathematical modeling, governing equations, boundary conditions, and the practical applicability of VSIs.

In [135], P-V and Q-V curves were plotted for multiple load buses using different load models to evaluate the system's maximum loadability limits. The voltage stability analysis was carried out using the continuation power flow method and validated on the IEEE 6-bus and 14-bus test systems.

VIII. DISCUSSIONS

Over the past several years, VSA has experienced tremendous developments, which have been brought to the fore by a change in the dynamics of systems attributed to the growing penetration of renewable energy sources, demand-side involvement, and real-time data technology. Research in the area of static voltage stability is based on the power flow equation, and this has developed significantly over time.

Although the concept of static analysis is still useful in determining the operating threshold of the power grid, it will not be an accurate representation of the underlying dynamics or the root cause of the voltage instability. Nonetheless, no matter the exact approach employed, the essence of it all is to determine the point where the power system is virtually close to its voltage instability threshold. Both the techniques discussed in this paper pay attention to various characteristics of the system to identify when the network is approaching or operating at its stability threshold. An example is the Jacobian Matrix obtained by the linearization of the equations of power flow. One Jacobian means that the system does not have a unique answer anymore, and it denotes the instability of voltages. In addition, the lowest single value of the Jacobian matrix also serves as a signal of proximity, which shows how a system is approaching a voltage collapse, especially in underconditioned systems.

Consequently, current VSA trends are moving towards data-driven and model-free. They include ML, AI, synchrophasor-based monitoring with the help of Phasor Measurement Units (PMUs), CPF, Modal Analysis, and SVD. The further development of ML, AI, and deep learning algorithms (e.g., SVM, ANN, CNN, LSTM) to VSA will eliminate the necessity of the full system models, allowing the direct use of data provided by real-time PMU (Phasor Measurement Unit). The methods are effective when the uncertainty and the nonlinearity are high in the system, which gives the ability to give quick evaluations that are appropriate to be used in online monitoring. High-speed (30-60 samples/sec) and time-synchronous grid-wide data provided by the combination of real-time measurements on WAMS and PMU provides the opportunity to detect early voltage instability and oscillatory states. The CPF of stronger and quicker predictor-corrector algorithms, such as parallel computers, will accurately find voltage collapse points at rising loads and minimize the computational time. Eigenvalue and singular value-advanced methods are used to trace weak buses and instability-sensitive modes, which provide useful information about vulnerable locations of the network. These techniques allow the most effective location of reactive power support devices, including FACTS devices, and would allow the proper determination of how close a system is to voltage collapse. The sensitivity and modal analyses are quite often utilized in the course of simulations, which adds to the overall flexibility of findings obtained through time-domain research. The new directions of studying voltage stability are also created with the help of these approaches. The Bifurcation theory provides a new point of view as it fills the gap between the fixed and dynamic analysis. By combining several approaches, the research on voltage stability becomes more efficient and flexible, with other approaches supplementing and reinforcing each other. Taken together, these innovations form a strong base for further study and development in the sphere of voltage stability of power systems.

IX. CONCLUSIONS

Voltage stability analysis is among the most important issues to be resolved for a stable and dependable electric power system. The study will help widen existing knowledge about the voltage stability assessment techniques. It is on this basis that the study illuminates some of the methods that have been used to provide more valid and reliable voltage stability measurements. It also discusses the issue of voltage instability and its effect on the stability of the entire power system. It also explores the challenges surrounding voltage instability and its impact on overall power system stability. The most traditional way of determining the margin of stability of voltages is by means of power flow equations. However, this approach is also not always efficient, as it does not determine exactly at what voltage the collapse of the voltage can occur. To overcome this drawback, more sophisticated methods are used, including Q-V sensitivity analysis, continuation power flow, voltage stability index, and participation factor analysis. These methods can be handy in determining the stability limits and determining the weakest sections of the network, including buses and branches that are of critical concern. They are also useful in providing the necessary level of reactive power compensation required to maintain voltage stability. The most recent trend in voltage stability measurement is towards the direction of more real-time, data-driven, and probabilistic determination. The newer approaches are in better positions to cope with the current issues of incorporating renewable energy, distributed generation, and system uncertainties. This modification is useful to enhance situational awareness and enable faster response; it also makes the power grid more resilient and stable.

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