

Dynamic Programming and Its Role in Optimal Exploitation of Financial Allocations for Maintenance of Production Lines

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Abstract— Our research presents a promising approach to the issue of efficiently allocating financial resources within the factory. We propose using dynamic programming to distribute these resources between different maintenance operations (preventive and remedial) for the machines and equipment that make up the factory's production lines. We aim to optimize production processes and enhance overall efficiency by doing so. The study's sample consists of the clothes factory located in Najaf. The main emphasis is on the production hall, namely the machines and equipment arranged in production lines (Layout). The findings indicate that dynamic programming can potentially address financial issues, particularly those related to the overall maintenance process and preventative and curative maintenance.

Keywords— Dynamic Programming, Financial Allocations, Maintenance Costs, Production Lines.

I. INTRODUCTION

Dynamic Programming (DP) is a crucial optimization technique that has significantly impacted various fields due to its ability to solve complex computational problems efficiently [1]. Initially coined by Richard Bellman in the 1950s, DP was designed to address multistage decision processes, particularly in optimal control problems where decisions and system states evolve over time [2]. Its core principle involves breaking down a problem into simpler subproblems, solving each subproblem once, and storing their solutions to avoid redundant computations, thus optimizing both time and space complexity [3]. DP's versatility is evident in its application across diverse domains such as bioinformatics, energy management, and medical fields, showcasing its robustness in handling various optimization challenges [4]. In investment and economic growth, DP aids in the optimal allocation of resources under uncertainty, enhancing decision-making processes in volatile environments [5]. Furthermore, DP's mathematical foundation allows it to address both deterministic and stochastic problems, providing feedback solutions through recursive methods like the Bellman equation [6]. Despite its computational intensity and space requirements, DP remains a powerful tool because it can yield globally optimal solutions, making it indispensable for solving sequential decision-making problems under uncertainty [2] [7]. Additionally, its integration into dynamic logic programming, as seen in languages like Epilog, further extends its utility in defining dynamic operations within logic

programming frameworks [8]. Overall, DP's broad applicability and effectiveness in optimizing complex problems underscore its importance in both theoretical and practical aspects of computer science. In administrative thought, especially within the management of operations, it has become known that machines and equipment within the production lines installed within the production hall are required continuously to carry out the maintenance process of both types of preventive and curative in an organized and programmed manner during certain periods specified and programmed in advance and this requires the management of operations in the organization to prepare a plan in advance and apply the scientific methods necessary to exploit the available financial resources within the framework of the pre-prepared allocations. To distribute them optimally, achieve the continuity of machinery and equipment work, and continue production lines. One of these scientific methods is dynamic programming, where the method puts in the hands of the decision-maker the necessary indicators to rationalize the process of exploiting the available resources and completing the maintenance process to achieve the continuity of the work of production lines and the continuation of production flow by the organization.

II. LITERATURE REVIEW

Dynamic programming is one of the tools of the quantitative approach to business management as well as an important tool for operations research [9]. The dynamic programming method is characterized by the entry of the time element in the mathematical model of the problem, unlike in linear programming or other types of mathematical programming models with fixed formulas, that is, under this method, the decision-maker can optimally plan processes that are subject to change or modification over time [10]. Therefore, dynamic programming is a mathematical or quantitative method used to address problems throughout the implementation phase of the problem-solving process. To reach the optimal method of change and modification required for the problem according to dynamic programming, the problem is divided into several consecutive stages, knowing that such problems are supposed to be originally divided into sequential time units while others are divided. To illustrate the idea of the mathematical formula of dynamic programming,

we assume that an industrial activity consists of N of the sequential stages, so at the beginning of the phase n (n=1,2,3,...,N) it is necessary to determine the value of the command to determine the value of the fundamental variable Xn. The series of fundamental changes of industrial activity as a whole should be selected in such a way that the criteria resulting from the determination of the function reach its maximum or minimum value, depending on the nature of the problem. Since the decision of one stage is made, the other stages are not neglected, i.e., the model is based on considering all the other stages.

In order to illustrate the idea of the mathematical model of dynamic programming and how it is used in general in the optimal distribution of financial allocations between production lines within the production hall, we assume that C is the number of monetary units that are supposed to be measured between N of production lines designed within the framework of the same production hall. Expected return from (i) from production lines (i=1,2,...,N) depends on the magnitude of the assignments that have been determined, that this relationship is expressed by the function (i=1,2,...,N)gi . Therefore:

(i=1,2,...,N)xi	Amount of specific allocations to the (i) production line
gi(xi) (i=1,2,...,N)	The amount of revenue obtained from (i) production lines

Therefore, the total amount of return from the production lines is calculated as follows

$$F(x_1, x_2, x_3, \dots, x_N) = g_1(x_1) + g_2(x_2) + g_3(x_3) + \dots + g_N(x_N)$$

where the resolution variables are: $x_1, x_2, x_3, \dots, x_N$:Which is supposed to meet the following conditions $x_1, x_2, x_3, \dots, x_N = C$

also: $x_i \geq 0 \quad i=1,2,\dots,N$

Solving these mathematical relationships will eventually result in the calculation of the values of the variables $x_1, x_2, x_3, \dots, x_N$

:which maximizes the value of the following target function

$$F(x_1, x_2, x_3, \dots, x_N) = g_1(x_1) + g_2(x_2) + g_3(x_3) + \dots + g_N(x_N)$$

:It fulfills the following condition

$$x_1 + x_2 + x_3 + \dots + x_N = C$$

$$x_i \geq 0$$

$$(i=1,2,3,\dots,N)$$

To solve a problem related to any productive business organization that has a production hall with a group of production lines, we assume that certain financial allocations are specified for N of the production lines, then for (N-1), and so on. Thus, problem analysis provides the conditions for applying dynamic programming, which means the dynamic steps of dividing financial allocations.

Giving (N) from the production lines an amount of (C) assignments of x_N leads to a yield of $g_N(x_N)$, knowing that x_N satisfies the following condition

$$0 \leq x_N \leq a_N$$

where; $a_N = C$

The remaining financial allocations of $a_{N-1} = a_N - x_N$

This amount is distributed in such a way that the yield obtained from the rest of the N-1 production lines is as high as possible.

The largest amount of yield from the remaining production lines N-1 is expressed by the symbol (

$$F_{N-1}(a_{N-1})$$

where; $F_{N-1}(a_{N-1}) = F_{N-1}(a_N - x_N)$

Since the beginning of the division of financial allocations x_N for a number of production lines of N will ensure an amount of returns: $g_N(x_N)$

Therefore, all financial returns from N production lines will amount to

$$g_N(x_N) + F_{N-1}(a_N - x_N)$$

The optimal size of x_N is that which maximizes the above target function. In addition to the above, the maximum possible returns obtained from N production lines are calculated as follows

$$F_N(a_N) = \text{Max} \{ g_N(x_N) + F_{N-1}(a_N - x_N) \}$$

$$0 \leq x_N$$

$$\leq a$$

The above mathematical relations were formulated, in which the rules of optimization were observed. If a similar model were formulated to determine the values of the variables, we would obtain an amount of returns from the set of production lines of N-1 depending on the size of the financial allocation of a_{N-1} : monetary unit, i.e

$$F_{N-1}(a_{N-1}) = \text{Max} \{ g_{N-1}(x_{N-1}) + F_{N-2}(a_{N-1} - x_{N-1}) \}$$

$$0 \leq x_{N-1} \leq$$

$$a_{N-1}$$

In the same way, for the x_{N-2} resolution variables, we have

$$F_{N-2}(a_{N-2}) = \text{Max} \{ g_{N-2}(x_{N-2}) + F_{N-3}(a_{N-2} - x_{N-2}) \}$$

$$0 \leq x_{N-2} \leq$$

$$a_{N-2}$$

Such a problem related to the division of financial allocations can be expressed graphically, as in Figure (1): which shows the variables of the mathematical model of dynamic programming during the total period of time.

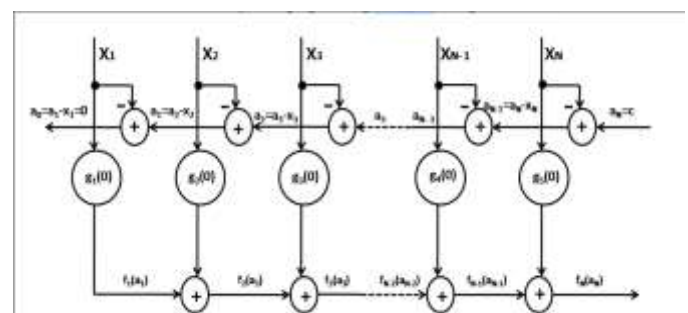


Figure 1. Movement of variables of the mathematical model of dynamic programming

Remedial and preventive maintenance in operations management:

Maintenance is one of the important engineering activities in the management of operations of continuity, which addresses the interruptions and damage caused to machines and equipment because of their use in the production process, as well as to maintain and maintaining a certain level of performance efficiency for these machines and equipment and their continuity in work in a manner consistent with the standard standards in this aspect. Maintenance is generally divided into two types as follows:

First: Preventive Maintenance:

It is that maintenance that begins before the formation of machinery and equipment, as it is represented by several procedures and operations, such as:

- 1) Lubrication and lubrication of machinery and equipment.
- 2) Spare parts are replaced according to a predetermined schedule.
- 3) Cleaning and auditing all the machine parts regarding operating requirements.

Second: Therapeutic Maintenance:

It is that maintenance that begins after a malfunction or stop in machinery and equipment and consists of some operations and activities, the most important of which are mentioned below:

1. Identify the causes of the malfunction and determine it accurately.
2. Replace all broken or damaged parts with new ones.
3. Remove all influences that may cause malfunction and stopping of machines and equipment in the coming period.

Whether maintenance is curative or preventive, such an engineering and operational activity requires the expenditure of cash from the specified allocations, and these amounts form an important part of the product's cost structure per unit.

Financial allocations for maintenance in the cost structure:

It is known that the cost structure per unit includes maintenance costs for machinery and equipment, in other words, the expenditures of financial resources under the heading of preventive and curative maintenance. They include the cost structure within the paragraph of direct or indirect expenses or what is known as operational expenses, as shown below:

TABLE I. allocations of maintenance costs.

Product cost structure	
xx	Direct raw materials
xx	Indirect raw materials
Xxx	
xx	Salaries and direct wages
xx	Salaries and indirect wages
Xxx	
xx	Direct industrial expenses Virtual 1 formula
xx	Indirect industrial expenses
Xxx	
xx	Administrative expenses
xx	Other expenses
Xxx	
(xxxx) Grand Total Cost Structure	

(¹) A.Jargowa, Rakhonk Kosztov, PwNTWa-v, تم اقتباس هذه الصيغة من كتاب: 2007, p.37

It is known that there is a direct relationship between maintenance expenses and costs (preventive and curative), which, whenever high, would be attached to the paragraph of industrial expenses in general and, thus, the total cost structure.

III. MODEL AND HYPOTHESES

The purpose of our research is to demonstrate that it is possible to apply the dynamic programming method to solve the problem of determining the optimal distribution of financial allocations that are available in the factory, which is the study sample, between the various maintenance operations (preventive and remedial) for the machines and equipment, which all represent the production lines of the factory, which is the study sample, and in a manner that results in the achievement of streamlined production processes.

The basic mathematical method used in this research is the dynamic programming method, which depends on what is available from the basic production requirements.

In order to bring the picture closer to the mind of the reader, we assume that the problem system consists in making a certain decision regarding the distribution of basic production inputs of a unit between N of the activities (n = 1,2,...,N), with each activity there is a result function (goal) associated with it, and symbolizes the said function that should lead us to the optimal state:

Whereas: $gn(Xn)$

XN The amount of basic supplies distributed to the activities.

Based on the above, a dynamic mathematical model can be formulated for the optimal distribution of basic production requirements, as follows:

It is required to specify positive values for the basic variables $X1, X2, \dots, Xn, \dots, XN$, (whereas: Xn is the quantity of basic requirements specified for activity n).

which maximizes the function of the sum of the results (objectives) of all activities (n=1,2,...,N).

:That is

$$F(X1, X2, \dots, Xn, \dots, XN) = g1(X1) + g2(X2) + \dots + gn(Xn) + \dots + gN(XN)$$

:that meet the following conditions

$$a = X1 + X2 + \dots + Xn + \dots + XN$$

From the conditions mentioned, it can be concluded that the quantity available in the basic necessities is specific.

Based on the above, we can conclude that under dynamic programming, the search for the optimal solution takes place in several stages according to the sequence of activities, where the optimal solution is determined at each stage. The decision taken at one activity stage is closely linked to the decision taken at the other stages.

The basic rule adopted in finding the optimal solution based on the dynamic programming model used from a particular system process is the following:

Regardless of the initial system status and the preliminary decision, the decisions should represent the optimal policy based on the situation resulting from the initial decision we imposed the following:

$St \leftarrow$ the state of the problem system at the moment t (the

situation in the previous state)

ST+1 ← system state problem at the moment t+1 (current situation)

DT+1 ← Resolution taken at the moment T+1 (current decision)

F ← the target function.

The above rule can be expressed mathematically as follows:
2 :

$$St+1=F(St+dt+1)$$

That is, the system's state at the moment is a function of the system's state at the previous moment (or the initial moment) with the decision made at the present moment or moment.

The following hypotheses were identified to accomplish the research:

First, Dynamic programming can be applied to address financial and operations management problems.

Second: Optimal exploitation of the financial allocations available for preventive and curative maintenance can be achieved using dynamic programming.

Third, dynamic programming effectively addresses financial and production problems as a mathematical operations research method.

IV. RESULTS

The men's clothing factory in Najaf Al-Ashraf is one of the factories of the General Company for the Manufacture of Ready-made Clothes affiliated with the Ministry of Industry and Minerals. This company bases its activity on several factories in addition to the men's clothing factory in Al-Najaf Al-Ashraf, as there is a factory in Sulaymaniyah for the production of women's clothing, and another factory in Mosul. For the production of children's clothing, the dishdasha factory in Anbar, and the clothing factory in Baghdad (tents factory).

He established the factory in Najaf Governorate with a capital of \$12,500,000, but the trial operation of the factory took place in 1986 and was limited to three production lines. The factory began activating its first production lines in March 1986, when the production quantity reached (63,166) pieces and after the success of the first activation process. The second revitalization process began in 1989, and after its success, all operating stages were implemented, the required technical personnel were trained, and attention was paid to securing primary and auxiliary materials.

The factory produces men's, women's, and children's clothing, in addition to military uniforms.

The factory is further identified by clarifying the organizational structure of the factory, according to which work and responsibilities are divided, authority is delegated, and relationships of familiarity and love are established between workers so that they can work together and produce with maximum efficiency to achieve the desired goals. The factory's organizational structure consists of a number of units that perform tasks. The basic functions of this organization, such as:

- production management.
- Engineering and services.

- Spare tools manufacturing committee.
- Financial Department.
- Administrative section.

The first and second departments (Production, Engineering and Services Department) are the basis for completing maintenance operations, in addition to other departments, such as the Financial Department, which provides the necessary financial allocations.

In order to simplify the idea of explanation of the factory under study, the focus is on the production hall, which explains the distribution of machines and equipment within the factory within the framework of the production lines, where the hall is divided into four groups of machines and equipment, which are:

- Group No. (1)/Production Line G1
- Group No. (2)/Production Line G2
- Group No. (3)/Production Line G3
- Group No. (4)/Production Line G4

The factory under study expanded production capacity by spending large amounts of money on building new production halls.

Note that the idea of developing and expanding the production capacity adopted by the factory is under study to modernize work and meet new demands on production, such as implementing uniform allowances, in addition to other obligations from allowances for holders of scientific titles and professors of universities and institutes. Therefore, it was decided to manage the factory under the supervision of the engineering staff in June 2001. Establishing a new production base with an area of 454 square meters to produce (300,000) pieces annually. The work included the following:

1. Preparing the designs and plans required for implementation.
2. Implementation of civil engineering works.
3. Implementing steam networks with a length of 224 meters, compressed air with a length of 125 square meters, air conditioning with a length of 165 meters, and suction with a length of 110 meters.

These expansions were carried out with all preventive and remedial maintenance requirements. The distribution of machines and equipment within the production hall according to a pre-prepared work system and rules called (layout).

Problem statements:

The necessary data was obtained about the problem in the men's clothing factory, and it relates to how to optimally distribute the available financial allocations between the four production lines, as the production hall consists of:

- Group No. (1) of machines and equipment G1
- Group No. (2) of machines and equipment G2
- Group No. (3) of machines and equipment G3
- Group No. (4) of machines and equipment G4

The production processes proceed from Group No. 1 and then to Group No. 4 as shown below:

As we mentioned previously, these production lines and the machines and equipment they contain require both types of maintenance (preventive and curative). The effectiveness of any production line depends on the size of the financial

(²) w.sadowsi, Programwania Dynamiczan, w-wa, Pwn, 2006. p. 31

allocations spent on these production lines. Accordingly, the data related to the effectiveness of each of the production lines in proportion to the size of the financial allocations spent is clear through the following table (2):

TABLE II. Amount of Financial Allocations Expected to be Disbursed on Each Production Line.

Periods per year								Size of financial allocations
7	6	5	4	3	2	1	0	
0,7	0,6	0,5	0,4	0,4	0,3	0,2	0,1	Production Lines .Production line No1 G1
0,7	0,7	0,6	0,6	0,5	0,3	0,3	0,2	.Production line No 2 G2
0,6	0,6	0,5	0,4	0,3	0,3	0,2	0,1	.Production line No 3 G3
0,5	0,5	0,5	0,3	0,3	0,2	0,2	0,1	Production line No. 4 G4

Note: The amounts are calculated in millions of dollars annually, and this was estimated by the Planning and Follow-up Department in the factory under study.

The highest value of the financial allocations that can be spent on maintenance in general cannot exceed the amount (7) million dollars, as the Operations Department, especially the Engineering Maintenance Division, how to distribute the planned amounts of cash for the purposes of maintenance (preventive and curative) in an optimal manner and in a way that achieves streamlined The production process is correct, so:

x_i = the amount of allocations specified to finance maintenance in general for all production lines in the production hall.

x_{2i} (i=1,2,3,4) The amount of allocations (basic variables) specified to finance maintenance in general for (i) production lines.

n = The level of effectiveness of production lines (machinery and equipment).

n_{2i} (i=1,2,3,4) The level of effectiveness of the production line (i).

From the data data, it is clear that:

$$0 \leq x_1 \leq 7$$

This idea can be treated as a problem divided into two stages

It becomes clear that (decision point No. 1) is divided into two parts, namely x_{11} and x_{12} (where: $x_1 = x_{11} + x_{12}$).

In the second step (step 2), especially at decision point No. (21) and also No. (22), the values x_{11} and x_{12} are divided into their corresponding values, which are x_{21} and x_{22} , as well as x_{23} and x_{24} , where:

$$x_{11} = x_{21} + x_{22}$$

$$x_{12} = x_{23} + x_{24}$$

The process of spending and disbursing financial resources on maintenance in general for the four production lines, corresponding to the values x_{21} , x_{22} , x_{23} , x_{24} , then the level of effectiveness that is obtained is:

$$n_{21}(x_{21}), n_{22}(x_{22}), n_{23}(x_{23}), n_{24}(x_{24})$$

Note that the effectiveness of the production lines shown in Figure No. () is calculated as follows:

$$n(x_1) = n_{21}(x_{21}) n_{22}(x_{22}) n_{23}(x_{23}) n_{24}(x_{24})$$

From the circumstances of the relationships and mathematical data above, it is clear that the amount of money spent on maintenance in general cannot exceed the figure of \$7 million, meaning that:

$$x_1 = x_{21} + x_{22} + x_{23} + x_{24} \leq 7$$

The process of solving such a problem requires, which leads to determining the maximum value of the following function:

$$n(x_1) = n_{21}(x_{21}) n_{22}(x_{22}) n_{23}(x_{23}) n_{24}(x_{24})$$

In this case, the following conditions are assumed to be met:

$$x_{21} + x_{22} + x_{23} + x_{24} \leq 7$$

and that:

$$x_{21} \geq 0, x_{22} \geq 0, x_{23} \geq 0, x_{24} \geq 0$$

According to what is shown in diagram No. (), the function $n(x_1)$ can be written as follows:

$$n(x_1) = n_{31}(x_{11}) n_{32}(x_{12})$$

whereas:

$$n_{31}(x_{11}) = n_{21}(x_{21}) n_{22}(x_{22})$$

$$n_{32}(x_{12}) = n_{23}(x_{23}) n_{24}(x_{24})$$

These above mathematical relationships, which together constitute the mathematical model for dynamic programming, are solved based on the form () and the rules of optimization, as follows:

First: Regarding Step No. 2 (Atap 2): It is clear that starting from decision point 21, where it is necessary to divide the amount into parts x_{21} , x_{22} in order to obtain the maximum effectiveness level for the production lines or 2, that is,:

$$n_{31}(x_{11}) = \max \{n_{21}(x_{21}) n_{22}(x_{22})\} = \max \{n_{23}(x_{23}) n_{22}(x_{22})\}.$$

For decision point 22, it is necessary to divide the amounts x_{24} and x_{23} , in order to obtain the maximum effectiveness level for production lines 3 and 4, that is:

$$n_{32}(x_{12}) = \max \{n_{23}(x_{23}) n_{24}(x_{24})\} = \max \{n_{23}(x_{23}) n_{24}(x_{24})\}.$$

Second: Regarding Step No. 1 (Atap 1): It is clear that starting No. 1 requires dividing the highest amount into the relationship $x_{11}+x_{12}$ to obtain the maximum possible level of effectiveness of machines and equipment in the production lines associated with this step, that is:

$$n(x_1) = \max \{n_{31}(x_{11}) n_{32}(x_{12})\} = \max \{n_{31}(x_{11}) n_{32}(x_{12})\}.$$

From Table No. (1) above, it is clear that decision variables consist of values and integers, and that this type of problem should be supported with figures and graphs. Carrying out the calculation process and following it up concerning decision point 21 and following the mathematical relationship:

$$n_{31}(x_{11}) = \max \{n_{21}(x_{21}) n_{22}(x_{22})\}.$$

From Figure No. (3) and the following figure No. (4), it is clear that on the horizontal axis there are the values $x_{21}, n_{21}, n_{21}(x_{21})$ and on the vertical axis there are the values $x_{22}, n_{22}(x_{22})$, noting that the values are $n_{21}(x_{21}) n_{22}(x_{22})$, obtained from the previous table No. (1).

For values related to the variables x_{21} , x_{22} that satisfy the following condition or value: $x_{21} + x_{22} \leq 7$

The following relationship is calculated: $n_{21}(x_{21}) n_{22}(x_{22})$. This is from Figure. (2) below:



Figure 2. The relationship between the values in the problem

The following condition is also met:

$$x_{21} + x_{22} = x_{11} = \text{const},$$

$$\text{Note that } x_{11} \leq 7$$

These values lie on the lines in the figure above that intersect at a point for these values, and on each line the values $n_{31}(x_{11}) = \max \{n_{21}(x_{21}) n_{22}(x_{22})\}$ are determined.

This relationship exists on the horizontal axis of Figure (3) below.

The calculations for the values at decision point 22 are carried out according to the relationship $n_{32}(x_{12}) = \max \{n_{23}(x_{23}) n_{24}(x_{24})\}$

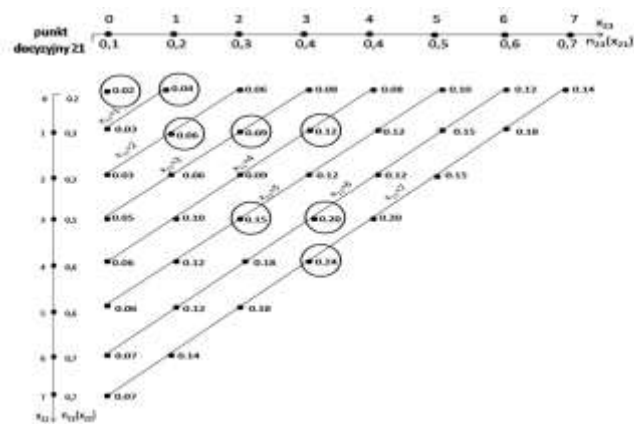


Figure 3. Relationship between variables according to the values given in the basic table

Similar mathematical operations are carried out as in the previous points. For decision points 21, clear results are obtained in Figure (7) below.

If a calculation process similar to what was mentioned above was carried out for decision points 21 and 22, in relation to decision point (1) and based on the relationship: $n(x_1) = \max \{n_{31}(x_{11}) n_{32}(x_{12})\}$,

Results shown in Figure (8) are obtained, where the values are: $n_{32}(x_{12})$ i $n_{31}(x_{11})$

This determines the maximum values on the straight lines shown in Figure (8) above

$$x_{21} + x_{22} = x_{11} = \text{const},$$

Likewise, from Figure No. (8), we notice the relationship: $x_{23} + x_{24} = x_{12} = \text{const},$

The results obtained are placed in the last table No. (2). For example, for the value $x_1 = 6$, from the last figure No. (9) we get the following value or result: $n(x_1) = \max \{n_{31}(x_{11}) n_{32}(x_{12})\} = 0.0054$

And the values:

$$x_{11} = 3, x_{12} = 3,$$

$$n_{31}(x_{11}) = n_{31}(3) = 0.09 \quad n_{32}(x_{12}) = n_{32}(3) = 0.06$$

This reading assumes the optimal value of values: $x_{11} = 3, x_{12} = 3$

On the basis of the previous figure (4), for the value $x_{12} = 3$, we have the following:

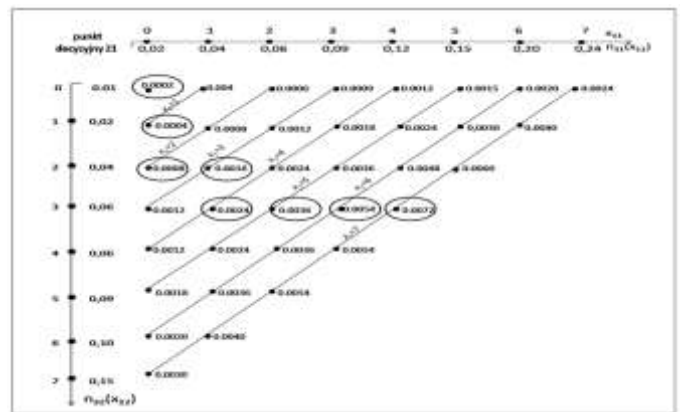


Figure 4. Calculating values for variables

$$n_{32}(x_{12}) = \max \{n_{23}(x_{23}) n_{24}(x_{24})\} = 0.06$$

$$x_{23} = 2, x_{24} = 1,$$

$$n_{23}(2) = 0.3 \quad n_{24}(1) = 0.2.$$

In the same way, from the previous figure (), for the value $x_{11} = 3$, we get the following:

$$n_{31}(x_{11}) = \max \{n_{21}(x_{21}) n_{22}(x_{22})\} = 0.09$$

$$x_{21} = 2, x_{22} = 1,$$

$$n_{21}(2) = 0.3, n_{22}(1) = 0.3.$$

Table (3) shows the results of the problem

TABLE III. The final results of the problem.

Effectiveness by production lines				Customizations distributed on production lines				Effectiveness Level	Financial allocations Calculated in \$
Line4 n24	Line3 n23	Line 2 n22	Line1 n21	Line1 x24	Line1 x23	Line1 x22	Line1 x21		
0,1	0,1	0,2	0,1	0	0	0	0	0,0002	0
0,1	0,2	0,2	0,1	0	1	0	0	0,0004	1
0,2	0,2	0,2	0,1	1	1	0	0	0,0008	2
0,2	0,2	0,2	0,2	1	1	0	1	0,0016	3
0,2	0,3	0,2	0,2	1	2	0	1	0,0024	4
0,2	0,3	0,3	0,2	1	2	1	1	0,0036	5
0,2	0,3	0,3	0,3	1	2	1	2	0,0054	6
0,2	0,3	0,3	0,4	1	2	1	3	0,0072	7

Table (3) shows the data representing the disbursement and spending program for financial allocations calculated in

dollars for seven time periods and with an increasing level of effectiveness (second column). The allocations are clearly distributed among the four production lines operating within the production hall in the ready-made clothing factory of the Holy Najaf Factory.

V. CONCLUSION AND DISCUSSION

Dynamic programming is a powerful tool for optimizing financial allocations in the maintenance of production lines, addressing the complexities of resource allocation and scheduling. It allows for the systematic division of investment expenditures to maximize productivity and minimize costs, as demonstrated in various studies. For instance, the application of dynamic programming in a manufacturing facility producing aluminum showed that maintaining machines at acceptable reliability values while minimizing maintenance costs can optimize the maintenance schedule effectively. Similarly, the integration of maintenance tasks into production scheduling using genetic algorithms has proven beneficial in handling the dynamic nature of modern production lines, ensuring that machines are maintained without disrupting production. The approach also extends to resource allocation problems, where dynamic programming helps in making multi-stage decisions to achieve maximum economic benefits. Moreover, the formulation of optimal shutdown policies for serial production lines using dynamic programming ensures that maintenance, calibration, and upgrades are performed efficiently during planned downtimes, balancing production goals with non-production tasks. Additionally, the use of real-time information and genetic algorithms in maintenance scheduling can significantly enhance system throughput and mitigate production losses. The Markovian approach to maintenance planning further supports the generation of optimal maintenance policies, demonstrating variability in the choice of optimal strategies based on workloads and costs. Furthermore, the fuzzy goal programming model for maintenance workforce optimization highlights the importance of reliable information for preventive and breakdown maintenance planning, ensuring operational productivity. Overall, dynamic programming and related optimization techniques provide a robust framework for the optimal exploitation of financial allocations in the maintenance of production lines, ensuring sustained productivity and cost efficiency.

The results show that dynamic programming can be applied to address financial problems, especially those related to the maintenance process in general and preventive and curative maintenance in particular. The optimal state can be achieved by applying the dynamic programming model and relying on the mathematical relationship:

$$St+1= F(St , dt+1)$$

The optimal return that is obtained is through the function:
 $F(x1,x2,x3,...,xN) = g1(x1) + g2(x2) + ... + gN(xN)$.

Dynamic programming, from Table (1), shows that it works to raise the level of effectiveness of machines and equipment through the optimal distribution of resources between the production lines operating in the production hall. Dynamic programming through the time periods shown in Table (2) provides decision-makers in the men's clothing factory with an effective means of rationalizing the process of distributing the financial allocations specified for this purpose. Therefore, the researchers recommend the following:

1. Moving towards applying quantitative methods, especially dynamic programming, in the process of distributing the financial allocations available for the maintenance process.
2. Adopting the quantitative indicators available in the last Table No. (2) in order to rationalize decisions on distributing the available financial allocations for preventive or curative maintenance purposes.
3. Avoid any distribution of financial allocations according to qualitative estimation, intuition, and guesswork.
4. Emphasizing all rationalization processes in financial allocation distribution decisions during the stages of applying dynamic programming.

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