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Response of Non-uniform Damped Beam to Mobile Distributed Loads on Variable Elastic Foundation

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Abstract—*An analytical procedure for the flexural motion of non-uniform structure on variable subgrade with moving weight is studied. The motion equation is solved by an assumed mode technique to obtain second order differential equation which is then solved by Laplace method and convolution theory. The effects of the variable elastic foundation, as well as damping intensity and torsional rigidity of the prismatic beam having moving distributed weight are assessed and the results displayed in plotted curves.*

Keywords— *Damping, Moving load, Non-uniform beam, Torsional rigidity, Variable foundation.*

I. INTRODUCTION

Owing to its significant relevance in structural and construction engineering, extensive works have been conducted to predict the behavioral pattern of beam-like on foundation subgrade traversed by the presence of load moving at uniform speed [1-4].

The appearances of damping in vibration of elastic structures have great effects and also useful for design engineering. Many authors have worked in this subject area for both beam and plate structural element. Crandall [5] studied the behavior of damping in role and special area where small amount of damping has an exaggerated importance in determining the dynamic behavior of a system are examined. Mousa and Reza [6] gave novel approach for free vibrational synthesis of the cracked cantilever beam having a breaking crack by taking into account the effect of the distributed structural damping. Robin and Rana [7] analyzed the vibrations of isotropic/orthotropic damped plate whose thickness vary and lying on foundation.

Practical problems in the structural dynamics especially under moving loads considers beam parameter to be vary. Here, the distribution of the non-uniform characteristics may be assumed as power function [8]. Also, when the structure has variable cross-section that is the beam parameters, mas and moment of inertia are considered as varying along the length of the structure [9, 10]. Gutierrez and Laura [11] presented dynamical analysis of a non-uniform cross-sectional structure traversed by concentrated load. In a later development, the vibrational behavior of a beam with cross-section beam having concentrated mass and force by FEM was considered by Ahmadian et al [12]. Taha [13] obtained a closed form solution for damped free vibration of a non-uniform shear beam resting on an elastic foundation.

Recent studies on distributed moving weight lying on elastic subgrade were addressed by authors [14-17]. Zhong et al [18] presented dynamic instability of a simply supported rectangular plate attached with the arbitrary concentrated masses owing to parametric resonance excited by an in-plane uniformly distributed periodic load along two opposite edges. Ogunyebi et al [19] examined vibration of non-prismatic beam-like lying on variable foundation subgrade by mobile concentrated forces. In the paper, the dynamic effect of rotatory inertia is neglected. Very recently, Ogunyebi [20] developed an analytical procedure for the solution of plate type structural members due to the influence of torsional rigidity and other vital parameters. The fourth order governing differential equation is addressed by the versatile method of Shadnam et al. In the work, a rise in the values of these structural parameters produces a noticeable effect on the critical velocity of the plate – type member.

In this present study, the effects of damping and torsional rigidity on variable non-uniform elastic foundation under moving distributed load is extensively studied to determine the dynamic role of the vital input characteristics in the motion equation.

II. THEORY AND FORMULATION

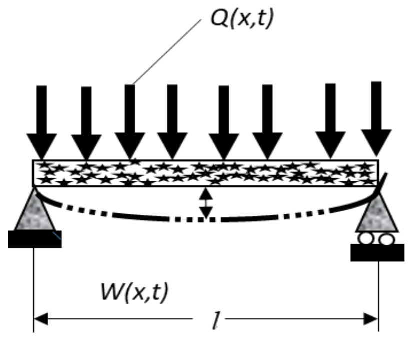

Fig. 1: Schematic diagram of moving distributed loads on non-uniform beam

Let us consider non-uniform thin beam moving with constant velocity lying on variable foundation subgrade and subjected to MDM system. Thus, vibration equation is

$$\frac{\partial^2}{\partial \tau^2}\left(EJ(\tau)\frac{\partial^2 \overline{W}(\tau,t)}{\partial \tau^2}\right) + \overline{m}^*(\tau)\frac{\partial^2 \overline{W}(\tau,t)}{\partial t^2}$$

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$$+D(\tau)\frac{\partial\bar{W}(\tau,t)}{\partial t}+2B\frac{\partial^2\bar{W}(\tau,t)}{\partial\tau^2}+K(\tau)\bar{W}(\tau,t)$$
$$=Q(\tau,t) \qquad (1)$$

with E as Young modulus, $J(\tau)$ as moment of inertia, $\bar{m}^*(\tau)$ as mass beam-like, $D(\tau)$ as damping coefficient, B as torsional rigidity $K(\tau)$ as foundation, $Q(\tau,t)$ as moving load, $\bar{W}(\tau,t)$ as transverse displacement, t as time, $\tau=x$ as spatial coordinate and $EJ(\tau)$ as stiffness.

The non-prestressed beam is simply supported thus, as in most practical case, the boundary conditions are assumed to be arbitrary i.e it can take different forms of classical BC's. Also, IC's of the problem has the form

$$\bar{W}(\tau,t)\Big|_{t=0}=0=\frac{\partial^2\bar{W}(\tau,t)}{\partial t^2}\Bigg|_{t=0}=0 \qquad (2)$$

As an example in this study, a variable elastic foundation of prestressed beam is considered [17]. To this end, the variable elastic foundation is given as

$$K(\tau)=K_0(4\tau-3\tau^2+\tau^3) \qquad (3)$$

where K_0 is variable foundation constant.

The load on the structural element is distributed and moving with uniform speed and chosen to be

$$Q(\tau,t)=PH(\tau-ct) \qquad (4)$$
$$J(\tau)=J_0(1+\beta\tau)^{m+2},\ \bar{m}^*(\tau)=\bar{m}_0^*(1+\beta\tau)^m,$$
$$D(x)=D_0(1+\beta\tau)^m \qquad (5)$$

Using equation (3), equation (4) and equation (5), equation (1), one becomes

$$EJ_0\frac{\partial^2}{\partial\tau^2}\left((1+3\beta\tau+3\beta^2\tau^2+\beta^3\tau^3)\frac{\partial^2\bar{W}(\tau,t)}{\partial x^2}\right)$$
$$+\bar{m}^*(1+\beta\tau)\frac{\partial^2\bar{W}(\tau,t)}{\partial t^2}+D_0(1+\beta\tau)\frac{\partial\bar{W}(\tau,t)}{\partial t}$$
$$+2B\frac{\partial^2\bar{W}(\tau,t)}{\partial\tau^2}+K_0(4\tau+3\tau^2+\tau^3)\bar{W}(\tau,t)=PH(\tau-ct)$$

III. METHOD OF SOLUTION

The motion equation governing the non-prismatic beam-like lying on variable foundation subgrade traversed by MDL (moving distributed load) is solved by the versatile Galerkin's method of obtaining approximate analytical solutions. The objective is to lower the order of the motion equation to 2nd order ODE. The solution method assumes the expansion of the unknown function $W(x,t)$ in a series of the orthonormal eigenfunction $\bar{y}_n(\tau)$ the choice of which must satisfy the boundary condition [18]. To this end, the method is given as

$$\bar{W}(\tau,t)=\sum_{n=1}^{\infty}V_n(t)\bar{y}_n(\tau) \qquad (7)$$

where $\bar{y}_n(\tau)$ are the mode shape functions and $V_n(t)$ the amplitude of the motion.

It can be shown that for simply supported non-prismatic beam, the modal shape of deflecting structure can be given as

$$\bar{y}_n(\tau)=\frac{\sin n\pi\tau}{L} \qquad (8)$$

Considering equations (7) and (8), equation (6) now becomes

$$\sum_{n=1}^{m}\Bigg\{EJ_0\Bigg(A_{n1}(\tau)\frac{n^4\pi^4}{L^4}\frac{\sin n\pi\tau}{L}+A_{n2}(\tau)\frac{n^3\pi^3}{L^3}\frac{\sin n\pi\tau}{L}$$
$$+A_{n3}(x)\frac{n^2\pi^2}{L^2}\frac{\sin n\pi\tau}{L}+2B\frac{n^2\pi^2}{L^2}\frac{\sin n\pi\tau}{L}+K_0(4\tau$$
$$+3\tau^2+\tau^3)\Bigg)V_n(t)+\bar{m}_0^*A_{n4}(x)\ddot{V}_n(t)\frac{\sin n\pi\tau}{L}$$
$$+D_0A_{n4}(x)\dot{V}_n(t)\frac{\sin n\pi\tau}{L}=PH(\tau-ct) \qquad (9)$$

The need to impose orthogonality condition in order to determine $V_n(t)$ is pertinent. Thus, the orthogonality of the function $\frac{\sin k\pi\tau}{L}$ in equation (9) gives

$$\int_0^L\sum_{n=1}^{m}\Bigg\{EJ_0\Bigg[A_{n1}(\tau)\frac{n^4\pi^4}{L^4}\frac{\sin n\pi\tau}{L}+A_{n2}(\tau)\frac{n^3\pi^3}{L^3}\frac{\sin n\pi\tau}{L}$$
$$+A_{n3}(x)\frac{n^2\pi^2}{L^2}\frac{\sin n\pi\tau}{L}+2B\frac{n^2\pi^2}{L^2}\frac{\sin n\pi\tau}{L}+K_0(4\tau$$
$$+3\tau^2+\tau^3)\Bigg]V_n(t)+\bar{m}_0^*A_{n4}(x)\ddot{V}_n(t)\frac{\sin n\pi\tau}{L}$$
$$+D_0A_{n4}(\tau)\dot{V}_n(t)\frac{\sin n\pi\tau}{L}\Bigg\}\cdot\frac{\sin k\pi\tau}{L}$$
$$=\int_0^L PH(\tau-ct)\frac{\sin k\pi\tau}{L} \qquad (10)$$

Equation (10) can be re-arranged to yield

$$Q_a(n,k)\ddot{V}_n(t)+Q_b(n,k)\dot{V}_n(t)+Q_c(n,k)V_n(t)$$
$$=PQ_d\frac{\sin k\pi\tau}{L} \qquad (11)$$

where

$$Q_a(n,k)=\bar{m}_0^*\int_0^L A_{n4}(\tau)\frac{\sin n\pi\tau}{L}\frac{\sin k\pi\tau}{L}d\tau \qquad (12)$$

$$Q_b(n,k)=G_{nn}\int_0^L A_{n4}(\tau)\frac{\sin n\pi\tau}{L}\frac{\sin k\pi\tau}{L}d\tau \qquad (13)$$

$$Q_c(n,k)=\int_0^L\Bigg\{EJ_0\Bigg[\Bigg(A_{n1}(\tau)\frac{n^4\pi^4}{L^4}+A_{n3}(\tau)\frac{n^2\pi^2}{L^2}\Bigg)\cdot\beta_{mm}$$
$$-A_{n2}(\tau)\frac{n^3\pi^3}{L^3}\frac{\cos n\pi\tau}{L}\frac{\sin k\pi\tau}{L}\Bigg]\Bigg\} \qquad (14)$$
$$+\left(2B\frac{n^2\pi^2}{L^2}\frac{\sin n\pi\tau}{L}+K_0(4\tau+3\tau^2+\tau^3)\right)\cdot\beta_{mm}d\tau\Bigg]$$

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$$Q_d = \frac{-n\pi}{L} \tag{15}$$

$$\beta_{mm} = \frac{\sin n\pi\tau}{L}\frac{\sin k\pi\tau}{L} \tag{16}$$

$$G_{nn} = D_0 A_{n4}(\tau) \tag{17}$$

Equation (11) is the 2nd order ODE with its constant coefficients. To proffer solution for equation (11), with aid of Laplace transformation, one obtains algebraic equation given as

$$\left[Q_a(n,k)S^2 + Q_b(n,k)S + Q_c(n,k)\right]V_n(s) = \frac{P_{nn}S}{S^2+\gamma^2} \tag{18}$$

where $\gamma = \frac{n\pi}{L}, P_{nn} = \frac{-k\pi}{L}$ (19)

Further simplification of equation (18) yields

$$V_n(s) = \frac{P_{nn}}{q_a+q_b}\left(\frac{1}{S-q_a} - \frac{1}{S-q_b}\right)\frac{S}{S^2+\gamma^2} \tag{20}$$

where

$$q_a = -Q_b + \sqrt{\frac{Q_b^{\ 2} - 4Q_aQ_c}{2Q_a}}, q_a = -Q_b - \sqrt{\frac{Q_b^{\ 2} - 4Q_aQ_c}{2Q_a}} \tag{21}$$

So that (20) gives,

$$V_n(s) = \left[\frac{q_a}{S+q_a}\cdot\frac{1}{q_a}\frac{S}{(S^2+\gamma^2)} - \frac{q_b}{S+q_b}\cdot\frac{1}{q_b}\frac{S}{(S^2+\gamma^2)}\right] \tag{22}$$

The inversion of equation (22), and using convolution theorem, one obtains

$$V_n(t) = \left[\frac{1}{q_a}\int_0^t e^{q_a t}G^{1*} - \frac{1}{q_b}\int_0^t e^{q_b t}G^{2*}\right] \tag{23}$$

where

$$G^{1*} = \int_0^t e^{-q_a u}\cos\gamma u du, G^{2*} = \int_0^t e^{-q_b u}\cos\gamma u du \tag{24}$$

Evaluating the integrals, equation (24) becomes

$$V_n(t) = P_{nn}\left[\frac{e^{q_a t}}{q_a(q_a^{\ 2}+\gamma)}\left(q_a - q_a e^{-q_a t}\cos\gamma t - \gamma e^{-q_a t}\sin\gamma t\right) - \frac{e^{q_b t}}{q_b(q_b^{\ 2}+\gamma)}\left(q_b - q_b e^{-q_b t}\cos\gamma t - \gamma e^{-q_b t}\sin\gamma t\right)\right] \tag{25}$$

Substituting equation (25) into equation (7), yields

$$\bar{W}(\tau,t) = \sum_{n=1}^{\infty} V_n(t)\frac{\sin n\pi\tau}{L} \tag{26}$$

equation (26) gives the vibration response to non-uniform beam-like lying on elastic subgrade of MDM at uniform speed.

IV. Numerical Results and Discussion

Numerical results for the non-prismatic beam problem is presented in this section. A non-uniform beam of length 12.192m and velocity 8.128m/s is considered. Other values used are modulus of elasticity 3.34×10^{10}N/m², moment of inertia $1.042 \times 10^4 m^4$ and mass beam is $3401.563 kg/m$.

Figure 1 depicts non-uniform beam-like lying on variable Winkler elastic subgrade traversed by moving distributed weight moving at uniform speed. From the figure, it is seen that for constant values of other important parameters, an increase in elastic subgrade K_0 decrease the profile of beam-like traversed by moving distributed weight at uniform speed.

Figure 2 depicts profile of non-uniform beam-like lying on variable Winkler subgrade traversed by moving distributed weight at uniform speed. Clearly, it is shown that higher values of the torsional rigidity B_0 lower the profile of the vibrating beam-like. Figure 3 depicts profile of non-uniform beam-like lying on variable Winkler subgrade traversed by moving distributed weight at uniform speed and observation from the figure shows that higher values of the damping parameter D_0 lower the profile of the vibrating beam-like element under MDL.

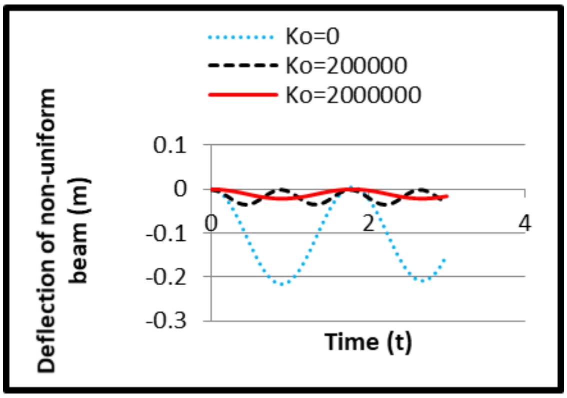

Fig. 1: Deflection of non-uniform structure at different values of K_0 and constant value of torsional rigidity B_0 and damping D_0

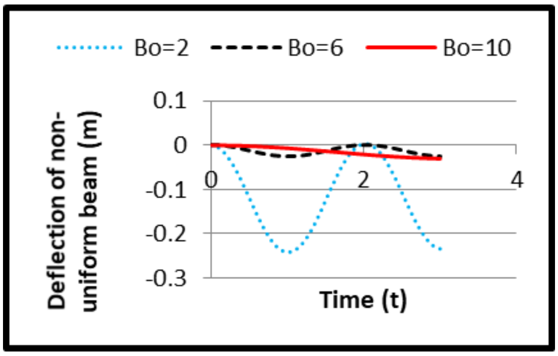

Fig. 2: Deflection of non-uniform structure at different values of B_0 and constant value of subgrade modulus K_0 and damping D_0

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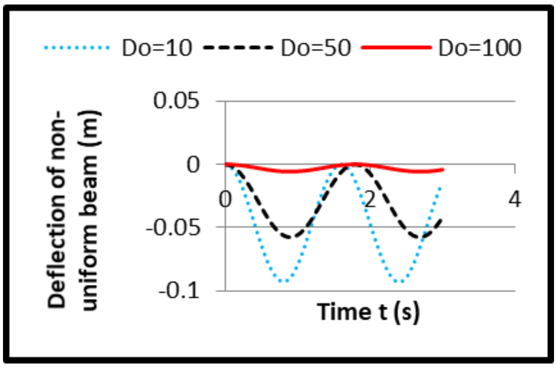

Fig. 3: Deflection of non-uniform structure at different values of D_0 and constant value of subgrade modulus K_0 and torsional rigidity B_0

V. Conclusion

Figure 2 depicts profile of non-uniform beam-like lying on variable Winkler subgrade traversed by moving distributed weight at uniform speed. Clearly, it is shown that higher values of the torsional rigidity B_0 lower the profile of the vibrating beam-like. Figure 3 depicts profile of non-uniform beam-like lying on variable Winkler subgrade traversed by moving distributed weight at uniform speed and observation from the figure shows that higher values of the damping parameter D_0 lower the profile of the vibrating beam-like element under MDL.

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