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Extended Abstract of Perishable Inventory Model for Postponed Demand with Reworks

Mohammad ATAULLAH
Assistant Professor of Mathematics, Directorate of Secondary and Higher Education, Ministry of Education, Bangladesh
Email address: ataul26@gmail.com

Abstract—*This paper analyzes an inventory model for perishable items with stochastic demand and items deteriorates with a constant rate θ. we consider an inventory system with rework where service warranty is provided for limited period of time. In most of the inventory models, a single stock is considered from where items are served for the customers. Here two stocks are considered. First one is for fresh items and the second one is for returned items. It is assumed that inventory level for the fresh and returned items are pre-determined and finite. When inventory level reaches at s a replenishment takes place with parameter γ. Arrivals of customers for fresh items form a Poisson process with parameter ξ. When inventory level reaches zero due to demands for fresh items, further demands are sent to a pool with maximum capacity W (<∞). Service for Pool's customer is level dependent and they will get service only when inventory level $I(t) \geq s+1$. The service time of each item are assumed to have independent exponential distributions. Return items after rework will be served with a parameter η. The joint probability distribution for inventory level of returned items and for fresh items are obtained in the steady state analysis. Some systems characteristics of the model are derived and the results are illustrated with the help of numerical examples.*

Keywords—*Perishable item, Postponed demand, Replenishment, Poisson process, Reworks.*
MSC 90B05, 90B30

I. INTRODUCTION

Returned management, for warranty claimed, is one of the key factor in the company's strategy of success. There are several reason behind the growth of return like as e-commerce, has increased the volume of product returns as customers are unable to see and touch the items they decide to buy, so they are more likely to return them. Most often the customers discovered that the product, they had bought through internet, did not have the functionality as they expected. On the other hand, in any stage of a production system, a certain number of defective items may produce due to various reason including poor production quality, material defects and subsequently a portion of them may be scrapped as well. In a production system where there is no repair or rework facility, defective items go to scrap as a result many industries having no recycling or reworking facility lose a big share of profit margin.

Many researchers have considered (s; S) inventory system with postponed demand and / or recycling process. An inventory model was developed by Krishnamoorthy and Islam (2004) where they analyzed an inventory system with postponed demand. A little attention was paid to the area of imperfect quality EPQ model with backlogging, rework and machine breakdown taking place in stock piling time. Chung (2011) developed a supply chain management model and presents a solution procedure to find the optimal production quantity with rework process. Chiu (2011) developed a Mathematical modeling for determining the replenishment policy for a EMQ model with rework and multiple shipments. Brojeswar Pal (2012) developed a multi-echelon supply chain model for multiple markets with different selling seasons and the manufacturer produces a random proportion of defective items which are reworked after regular production and are sold in a lot to another market just after completion of rework. Krishnamoorthy (2013) developed a single stage production process where defective items produced are rework and two models of rework processes are considered, an EPQ without shortages and with shortages Sivashankari and Panayappan (2014) proposed a Production inventory model where they consider reworking of imperfect production, scrap and shortages. Mohammad Ekramol Islam (2017) developed two inventory system with reworks facilities these are the base of our study.

II. MATHEMATICAL MODEL

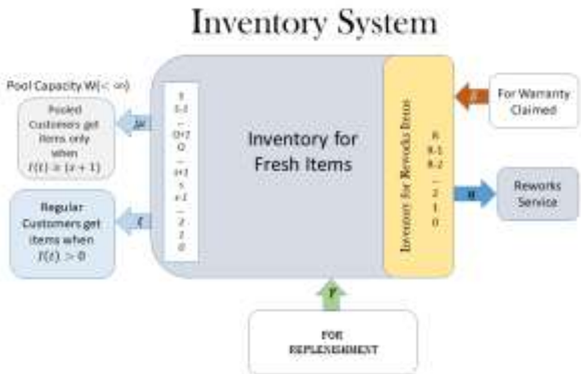

Fig. 1. Perishable Inventory for postponed demand with reworks.

(a) Assumptions:

Initially the inventory level for fresh items is S and for return items is R. Maximum capacity of the pool is W. Arrival rate of demands follows poisson process with parameter ξ, $j\mu$ and δ for fresh items, pool customers and return items respectively. Lead-time is exponentially distributed with parameter γ for fresh items.If the inventory of return items is at R, then home service will be provided as early as possible with exponential parameter μ.When inventory level

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$I(t) \geq s+1$ either external or pooled customer's are served. When inventory level $I(t) < s$ only external demand are met.Fresh items will decay in a constant rate θ.

(b) Notations:

S→Maximum inventory level for fresh items. R →Maximum inventory level for returned items. W →Maximum Capacity of the pool. ξ → Arrival rate of demands for fresh items. $j\mu$ → Arrival rate of pooled customer for fresh items. δ →Arrival rate of demands for returned items. γ → Replenishment rate for fresh items. η → Service rate for returned items. θ → Deterioration rate of fresh items. $I(t)$ →Inventory level at time t for fresh items. $X(t)$ → Inventory level at time t for retuned items. $F(t)$ →Number of pooled customer at time t. $E = E_1 \times E_2 \times E_3$ → The state space of the process. Where,E_1={0, 1, 2, … , S},E_2={0, 1, 2, … , R } and E_3={0, 1, 2, … , W } ;$e_{(R+1)\times(W+1)} = (1,1,1,\dots,1)'$; an (R +1) $\times (W+1)$-Components column vector of 1's.

(c) Model Analysis:

In our model, we have fixed maximum inventory level for fresh items at *S*, for return items at *R* and maximum capacity of pool is *W* .The inter-arrival time between two sucessive demands are assume to be exponentially distributed with parameter ξ for fresh items, δ for return items and $j\mu$ for pooled customer. Each demand is for exactly one unit for each items. When inventory level reduced to s an order for replenishment is placed. Lead-time is exponentially distributed with parameter γ. If inventory level $I(t) \geq s+1$ either external customer or pool customer will be served otherwise only external customer will be served When inventory level for the return items reached at *R* home service will be provided as soon as possible.

Now, the infinitesimal generator of the three dimensional markov process $\{I(t), X(t), F(t); t \geq 0\}$ can be defined as $\tilde{A} = \big(a(i,j,k,l,m,n)\big); (i,j,k),(l,n,m) \in E$

Hence, we get

$$\tilde{A} = \begin{cases} (\xi+\theta) & : i = 1,2,\dots,S, l = i-1;\ j = 0,1,2,\dots,W, m = j;\ k = 0,1,\dots,R, n = k \\ j\mu & : i = 1,2,\dots,S, l = i-1;\ j = 0,1,2,\dots,W,\ m = j;\ k = 0,1,\dots,R, n = k \\ \delta & : i = (s+1),(s+2),\dots,S, l = i-1;\ j = 0,1,2,\dots,W, m = j; k = 0,1,\dots,R-1, n = k+1 \\ \eta & : i = (s+1),(s+2),\dots,S, l = i-1;\ j = 0,1,2,\dots,W, m = j;\ k = 1,2,\dots,R, n = k-1 \\ -(\xi+\theta+m\mu+\eta) & : i = (s+1),(s+2),\dots,S, l = i-1;\ j = W, m = j \dots k = R, n = k \\ -(\xi+\theta+m\mu+\eta+\delta) & : i = (s+1),(s+2),\dots,S, l = i-1;\ j = W, m = j \dots k = 0,1,2,\dots,R-1, n = k \\ -(\xi+\theta+m\mu+\delta) & : i = (s+1),(s+2),\dots,S, l = i-1;\ j = W, m = j \dots k = 0, n = k \\ (\xi+\theta) & : i = 0, l = i;\ j = 0,1,2,\dots,W-1, m = j+1;\ k = 0,1,\dots,R, n = k \\ \delta & : i = 0,1,2,\dots,s, l = i;\ j = 0,1,2,\dots,W, m = j; k = 0,1,\dots,R-1, n = k+1 \\ \eta & : i = 0,1,2,\dots,s, l = i;\ j = 0,1,2,\dots,W, m = j;\ k = 1,2,\dots,R, n = k-1 \\ -(\eta+\gamma) & : i = 0, l = i;\ j = W, m = j;\ k = R, n = k \\ -(\delta+\eta+\gamma) & : i = 0, l = i;\ j = W, m = j;\ k = 1,2,\dots R-1, n = k \\ -(\delta+\gamma) & : i = 0, l = i;\ j = W, m = j;\ k = 0, n = k \\ -(\xi+\theta+\delta+\gamma) & : i = 0, l = i;\ j = 0,1,\dots,W-1, m = j;\ k = 1,2,\dots R-1, n = k \\ -(\xi+\theta+\delta+\gamma) & : i = 0,1,2,\dots,s, l = i;\ j = 0,1,2,\dots,W, m = j;\ k = 0, n = k \\ \gamma & : i = 0,1,2,\dots,s, l = i+Q; j = 0,1,2,\dots,W, m = j; k = 0,1,2,\dots,R, n = k \\ -(\xi+\theta+\eta+\gamma) & : i = 0,1,2,\dots,s, l = i; j = 0,1,2,\dots,W, m = j; k = R, n = k \\ -(\xi+\theta+\eta+\delta+\gamma) & : i = 0,1,2,\dots,s, l = i; j = 0,1,2,\dots,W, m = j; k = 1,2,\dots,R-1, n = k \\ 0 & : \mathit{Otherwise} \end{cases}$$

So, we can write the partioned matrx as follows:

$$\tilde{A} = \begin{cases} A_1 \text{ if } l = i-1, i = (s+1),(s+2),\dots,S \\ A_2 \text{ if } l = i, i = (s+1),(s+2),\dots,S \\ A_3 \text{ if } l = i, i = 1,2,\dots,s \\ A_4 \text{ if } l = i, i = 0 \\ A_5 \text{ if } l = i-1, i = 1,2,\dots,s \\ A_6 \text{ if } l = i+Q, i = 0,1,2,\dots,s \end{cases}$$

III. Stability Analysis of the System

It can be seen from the structure of matrix $\tilde{A}$ that the state space *E* is irreducible. Let the limiting distribution be denoted by $\pi^{(i,j,k)}$:

$\pi^{(i,j,k)} = \lim_{t\to\infty} \Pr[I(t), F(t), X(t) = (i,j,k)], (i,j,k) \in E$

Since the limiting distribution exist that satisfies the following equations:

$$\pi\tilde{A} = 0 \dots (1a) \text{ and}$$

$$\sum_{i=0}^{S}\sum_{j=0}^{W}\sum_{k=0}^{R}\pi^{(i,j,k)} = 1 \dots (1b).$$

Solving the equations (1a), Satisfying the equation (1b) , we get the desired state probability vectors $\pi^{(i,j,k)}$.

IV. Systems Characteristics

(a) Mean inventory level: (i) Mean inventory level for fresh items $L_1 = \sum_{i=1}^{S} i \sum_{j=0}^{W}\sum_{k=0}^{R}\pi^{(i,j,k)}$

(ii) Mean inventory level for Return items

$L_2 = \sum_{k=1}^{R} k \sum_{j=0}^{W}\sum_{i=0}^{S}\pi^{(i,j,k)}$

(b) Re-order rate for fresh items

$R_o = (\xi+\theta)\sum_{j=0}^{W}\sum_{k=0}^{R}\pi^{(s+1,j,k)}$

(c) Average customer lost to the system

$C_l = \xi\sum_{j=0}^{W}\sum_{k=0}^{R}\pi^{(0,j,k)}$

(d) Service completion rate for return items

$S_r = \mu\sum_{i=0}^{S}\sum_{j=0}^{W}\sum_{k=1}^{R}\pi^{(i,j,k)}$

(e) Expected number of pooled customer

$C_p = \sum_{i=0}^{S}\sum_{j=1}^{W} j\sum_{k=0}^{R}\pi^{(i,j,k)}$

(f) Expected Total Cost (ETC)
$= c_1L_1 + c_2L_2 + c_3R_o + c_4C_l + c_5S_r + c_6C_p$
Where, $c_1 =$ Holding cost per Unit for fresh items,
c_2=Holding cost per Unit for return items,
c_3= Replenishment cost per order,
c_4 = Cost for per unit lost sales,
c_5= Servicing cost per unit for return items and
c_6 = Waiting cost per customer.

V. NUMERICAL ILLUSTRATION

PUTTING, S=5, s=2, W=3, R=2,Q=3, ξ=0.30, δ=0.02, γ=0.40, $\eta = 0.1$, μ=0.3, θ=0.15 c_1=1.25, c_2=1.30, c_3=1.45, c_4=2.50, c_5=1.50 AND $c_6 = 1.75$
we get,

TABLE I. Numerical values of the characteristics of the Inventory system.

L_1	L_2	R_o	C_l	S_r	C_p	ETC
3.67536	0.193548	0.128702	0.030226	0.069231	0.240288	5.64202

VI. DISCUSSION

It is cleared that all costs related to inventory system increase total cost but increment of one unit holding cost for fresh item increase total cost more rapidly than that of rework items. On the other hand servicing cost per rework items increase total cost but for the shake of business goodwill product warranty is a must. Ordering cost per order increase total cost slightly but cost per pooled customer raises total cost in a higher rate. So servicing rate for both regular customer and pooled customer should be increased.

VII. CONCLUSIONS

In the open market system, any production organizations need to consider postponed demand and product warranty for shake of their goodwill as well as to compete with others. In such a situation, the model is more appropriate to take plan for their future activities.

VIII. ACKNOWLEDGMENT

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Appendix

TABLE II. State probability vectors of the system.

$\pi^{(5,3,2)}$	0.0000184178	$\pi^{(4,1,2)}$	0.0004309821	$\pi^{(2,3,2)}$	0.0000621602	$\pi^{(1,1,2)}$	0.0003125262
$\pi^{(5,3,1)}$	0.0000920892	$\pi^{(4,1,1)}$	0.0021549106	$\pi^{(2,3,1)}$	0.0003108010	$\pi^{(1,1,1)}$	0.0015626308
$\pi^{(5,3,0)}$	0.0004604459	$\pi^{(4,1,0)}$	0.0107745530	$\pi^{(2,3,0)}$	0.0015540052	$\pi^{(1,1,0)}$	0.0078131540
$\pi^{(5,2,2)}$	0.0000942460	$\pi^{(4,0,2)}$	0.0064050572	$\pi^{(2,2,2)}$	0.0002473958	$\pi^{(1,0,2)}$	0.0024125395
$\pi^{(5,2,1)}$	0.0004712300	$\pi^{(4,0,1)}$	0.0320252859	$\pi^{(2,2,1)}$	0.0012369788	$\pi^{(1,0,1)}$	0.0120626973
$\pi^{(5,2,0)}$	0.0023561501	$\pi^{(4,0,0)}$	0.1601264296	$\pi^{(2,2,0)}$	0.0061848939	$\pi^{(1,0,0)}$	0.0603134866
$\pi^{(5,1,2)}$	0.0003148412	$\pi^{(3,3,2)}$	0.0001174137	$\pi^{(2,1,2)}$	0.0005903272	$\pi^{(0,3,2)}$	0.0003783952
$\pi^{(5,1,1)}$	0.0015742058	$\pi^{(3,3,1)}$	0.0005870686	$\pi^{(2,1,1)}$	0.0029516359	$\pi^{(0,3,1)}$	0.0018919759
$\pi^{(5,1,0)}$	0.0078710292	$\pi^{(3,3,0)}$	0.0029353431	$\pi^{(2,1,0)}$	0.0147581797	$\pi^{(0,3,0)}$	0.0094598796
$\pi^{(5,0,2)}$	0.0040506835	$\pi^{(3,2,2)}$	0.0002324756	$\pi^{(2,0,2)}$	0.0045570190	$\pi^{(0,2,2)}$	0.0004551644
$\pi^{(5,0,1)}$	0.0202534177	$\pi^{(3,2,1)}$	0.0011623782	$\pi^{(2,0,1)}$	0.0227850950	$\pi^{(0,2,1)}$	0.0022758220
$\pi^{(5,0,0)}$	0.1012670887	$\pi^{(3,2,0)}$	0.0058118912	$\pi^{(2,0,0)}$	0.1139254748	$\pi^{(0,2,0)}$	0.0113791098

$\pi^{(4,3,2)}$	0.0000158899	$\pi^{(3,1,2)}$	0.0008050949	$\pi^{(1,3,2)}$	0.0000329083	$\pi^{(0,1,2)}$	0.0008655890
$\pi^{(4,3,1)}$	0.0000794495	$\pi^{(3,1,1)}$	0.0040254747	$\pi^{(1,3,1)}$	0.0001645417	$\pi^{(0,1,1)}$	0.0043279446
$\pi^{(4,3,0)}$	0.0003972475	$\pi^{(3,1,0)}$	0.0201273735	$\pi^{(1,3,0)}$	0.0008227086	$\pi^{(0,1,0)}$	0.0216397229
$\pi^{(4,2,2)}$	0.0001060728	$\pi^{(3,0,2)}$	0.0080709726	$\pi^{(1,2,2)}$	0.0001309742	$\pi^{(0,0,2)}$	0.0015509182
$\pi^{(4,2,1)}$	0.0005303640	$\pi^{(3,0,1)}$	0.0403548629	$\pi^{(1,2,1)}$	0.0006548711	$\pi^{(0,0,1)}$	0.0077545911
$\pi^{(4,2,0)}$	0.0026518202	$\pi^{(3,0,0)}$	0.2017743145	$\pi^{(1,2,0)}$	0.0032743556	$\pi^{(0,0,0)}$	0.0387729557